Restoring phase coherence in 1d superconductivity by power-law hopping

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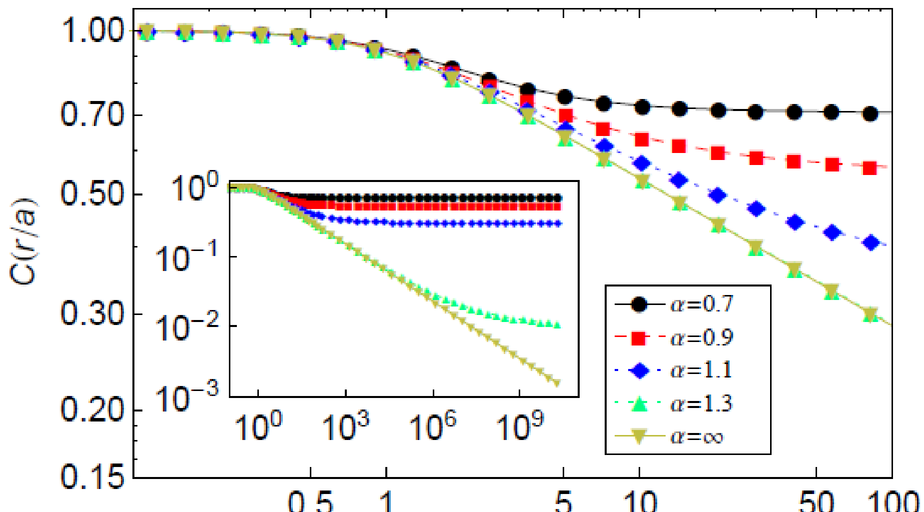
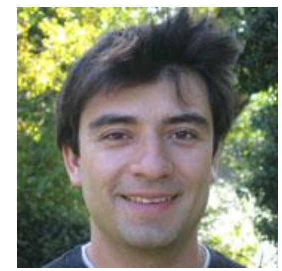
http://www.tcm.phy.cam.ac.uk/~amg73/

Tezuka
Kyoto





Lobos
Maryland

FCT
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MARIE CURIE ACTIONS

EPSRC
Engineering and Physical Sciences
Research Council

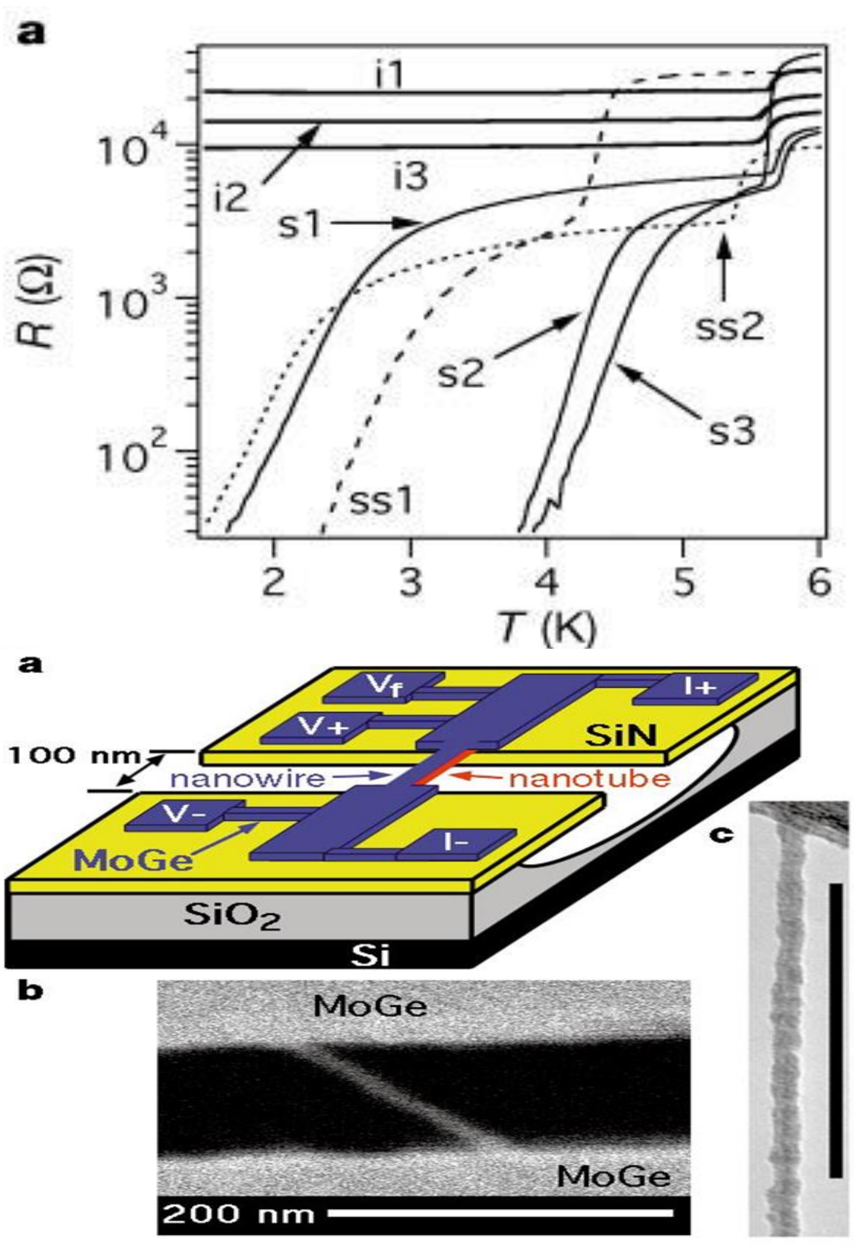
Tinkham, Arutyunov

Mermin-Wagner theorem

Low dimensions = No ***true*** superconductivity

Quantum fluctuations

Thermal Fluctuations



$$|\Delta(\mathrm{r},\mathrm{t})|e^{i\theta(r,t)}$$

Fluctuation

$\Delta(r_0, t_0) \approx 0$

Phase-slips

$\theta \approx 0 \rightarrow 2\pi$

Finite Resistance

$R \propto e^{-S_{inst}}$

Thermal

Langer & Ambegaokar, PR. 164, 498 (1967).
McCumber & Halperin PRB 1, 1054 (1970).

Kosterlitz Thouless transition

Large Instantons

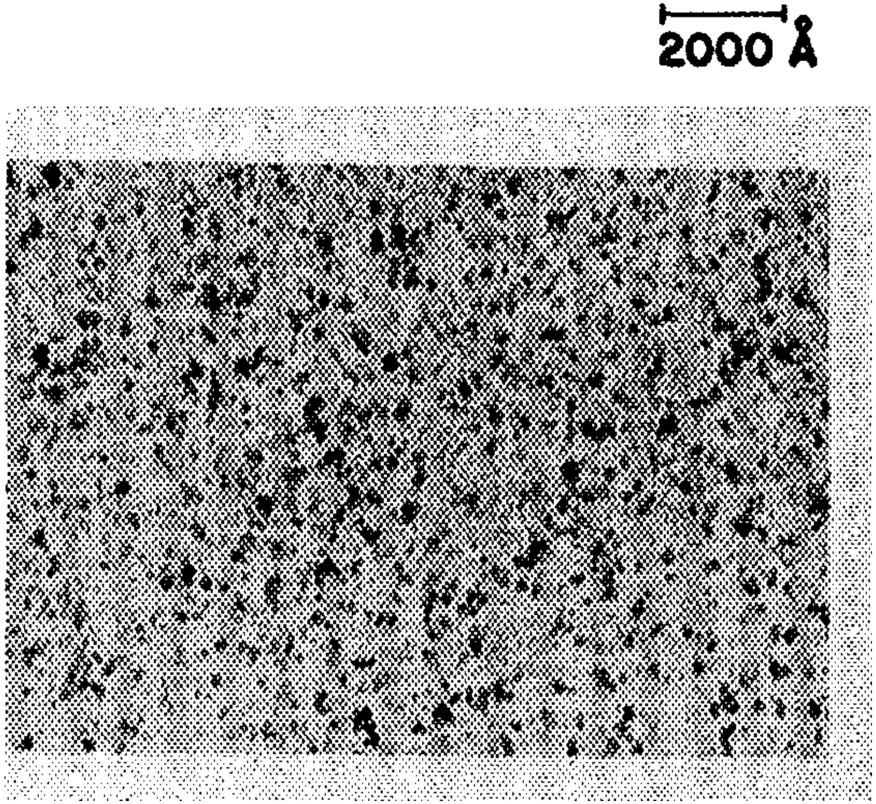
Quantum

Zaikin, A. D., Golubev, et al, PRL 78, 1552 (1997).

$T_c^{KT} < T_c^{MF}$

Early Experiments

Metal	T_c (°K)	T_c/T_{c0}	d (Å)	ρ_0
Al	3.0	2.6	40	0.19
Ga	7.2	6.5	⋯	0.20
Sn	4.1	1.1	110	0.31
In	3.7	1.1	110	0.36
Pb	7.2	1.0	⋯	0.53



Abeles, Cohen, Cullen, Phys. Rev. Lett., 17, 632 (1966)

Crow, Parks, Douglass, Jensen, Giaver, Zeller....

A.M. Goldman, Dynes, Tinkham…

90's

Thinner

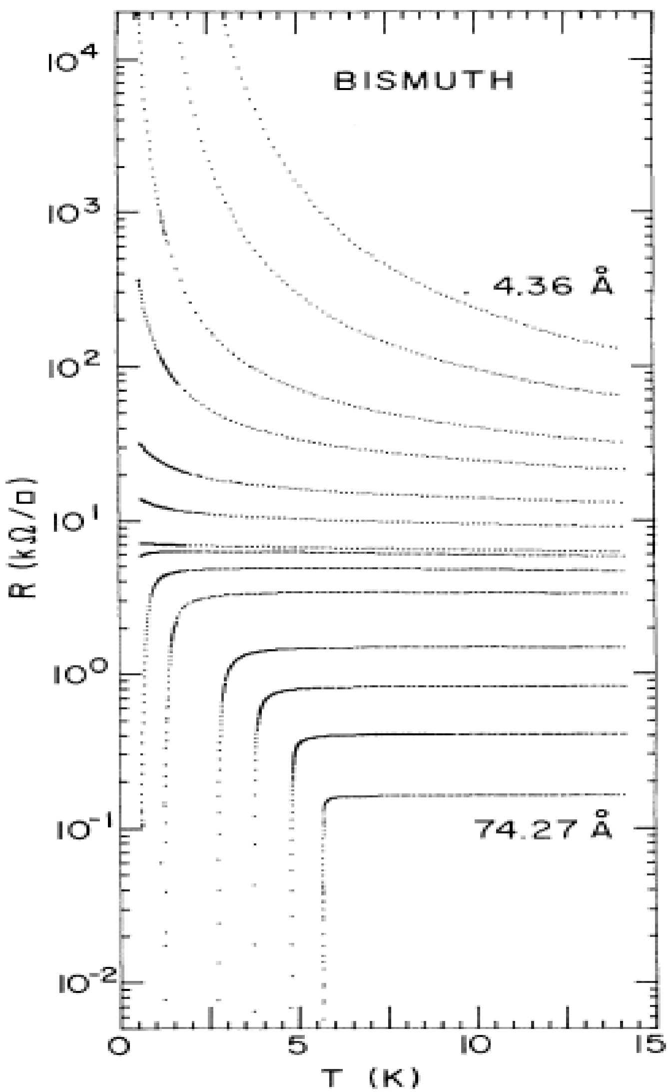
Smoother

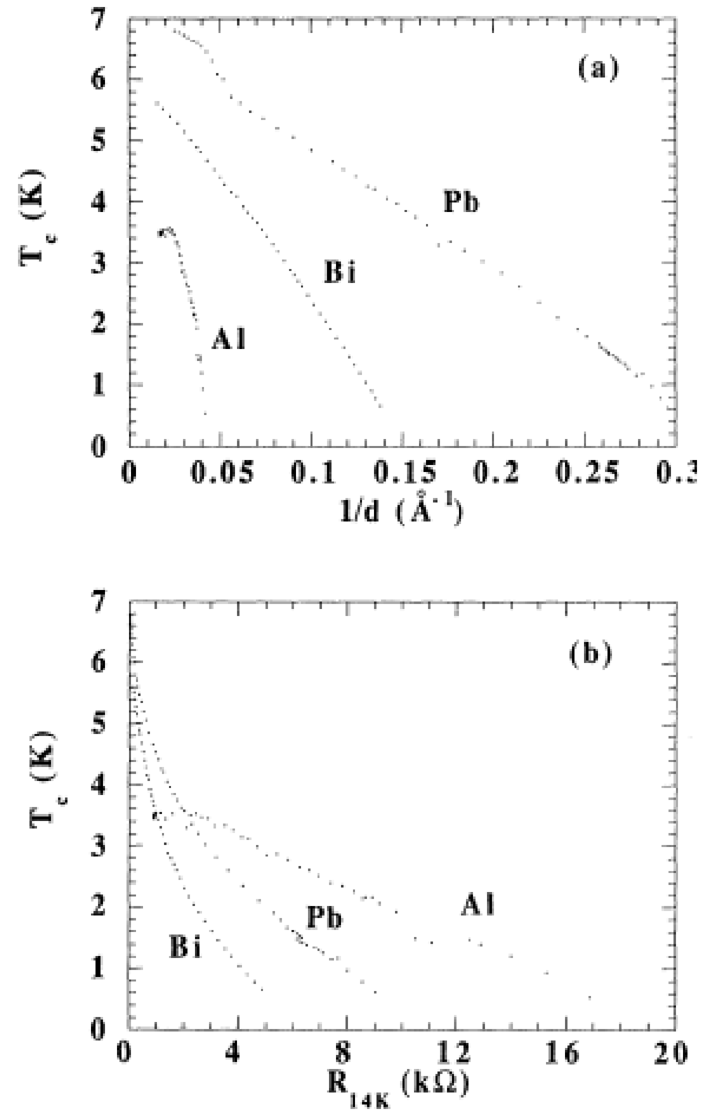
BKT

Transition

$R_N > R_q$

Vortices unbinding

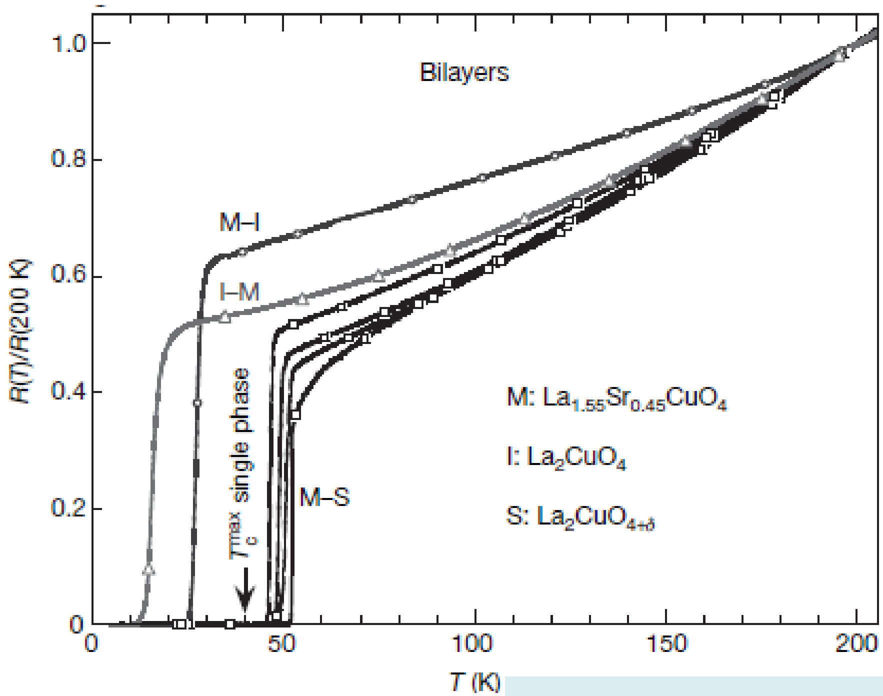


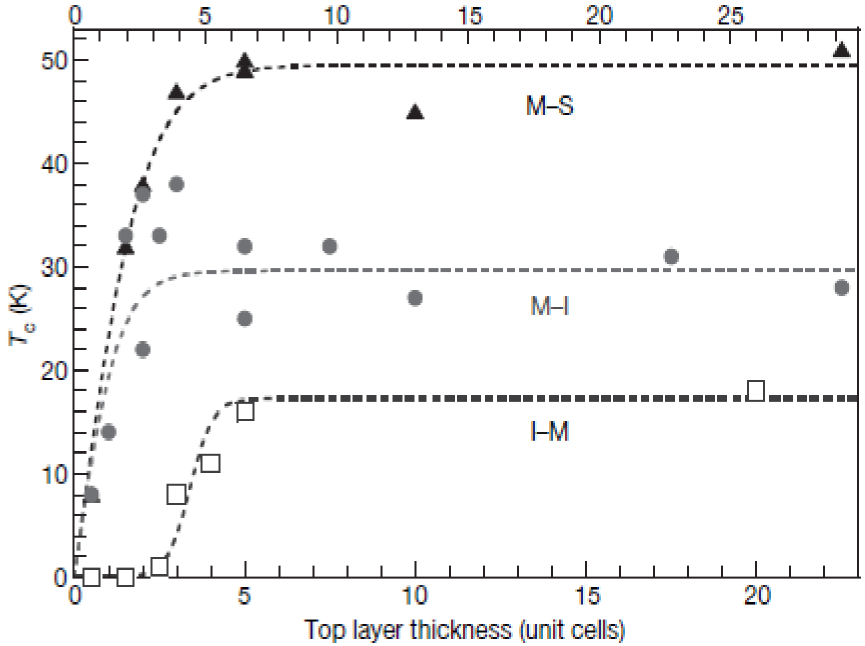


A.M. Goldman et al.

PRL 62 2180 (1989)
PRB 47 5931 (1993)

Cuprates high T_c Heterostructures

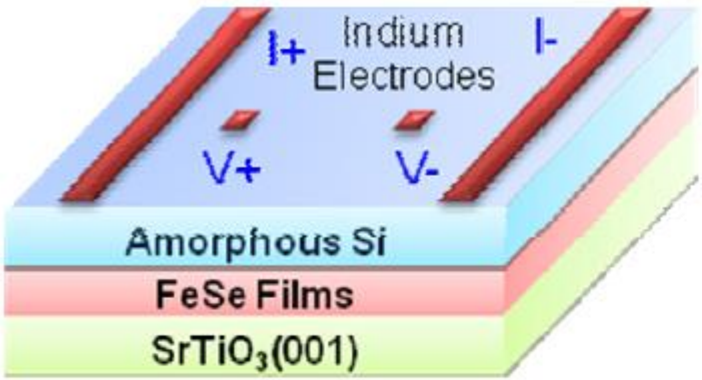


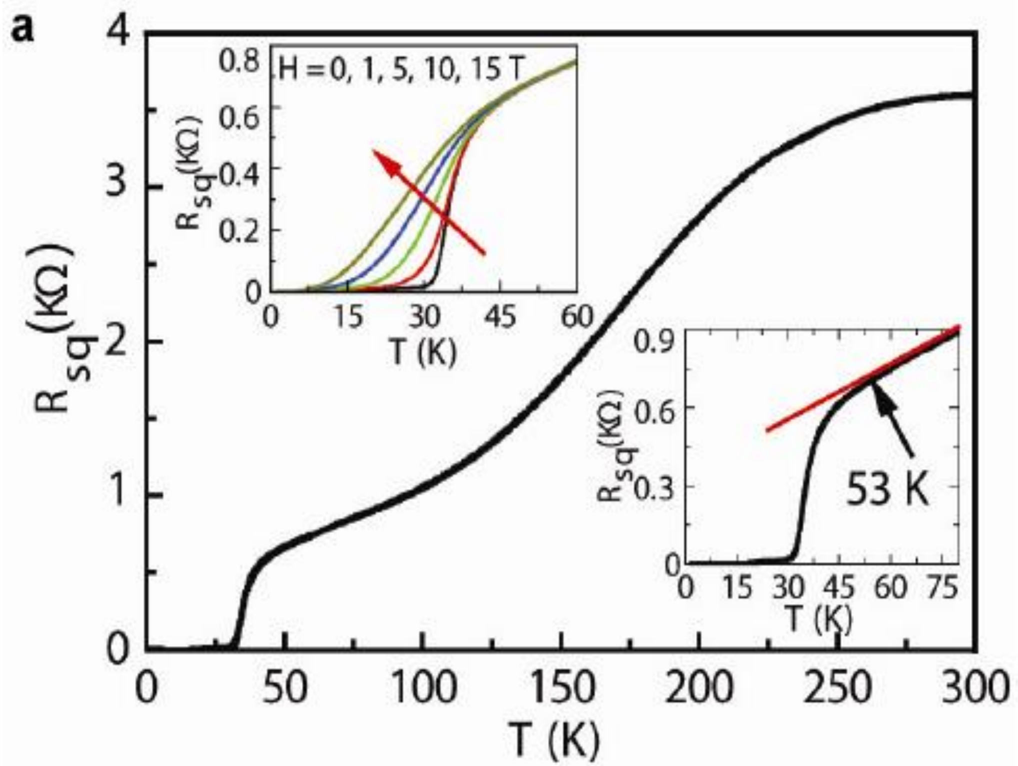


Bozovic et al., Nature 455, 782 (2008)

Higher T_c!!

Iron Pnictides Heterostructures

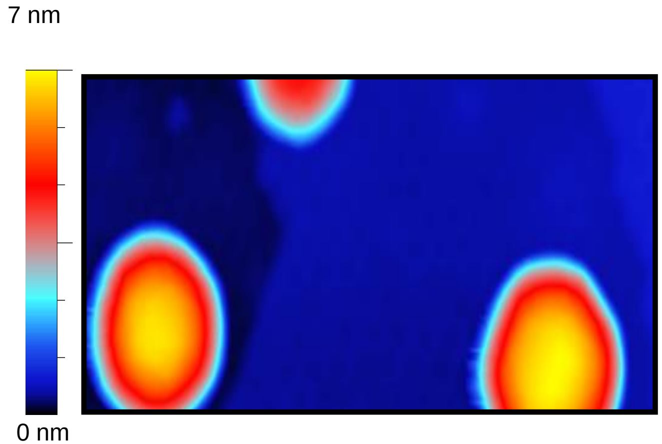


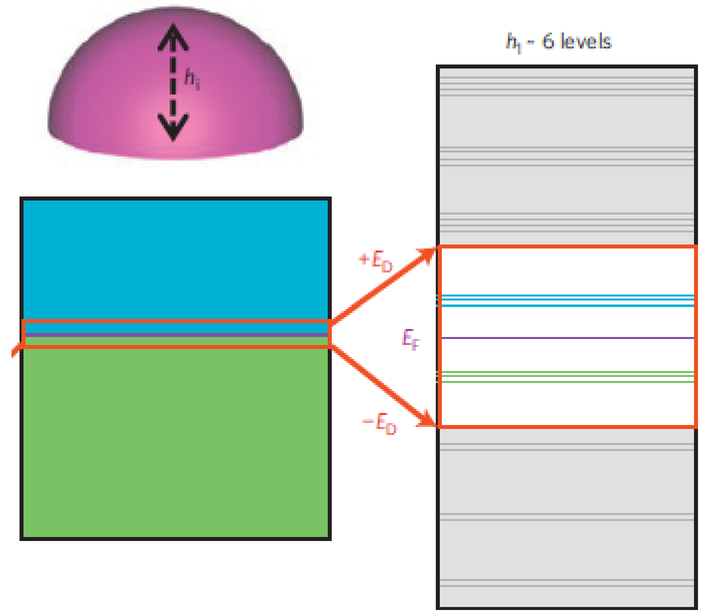


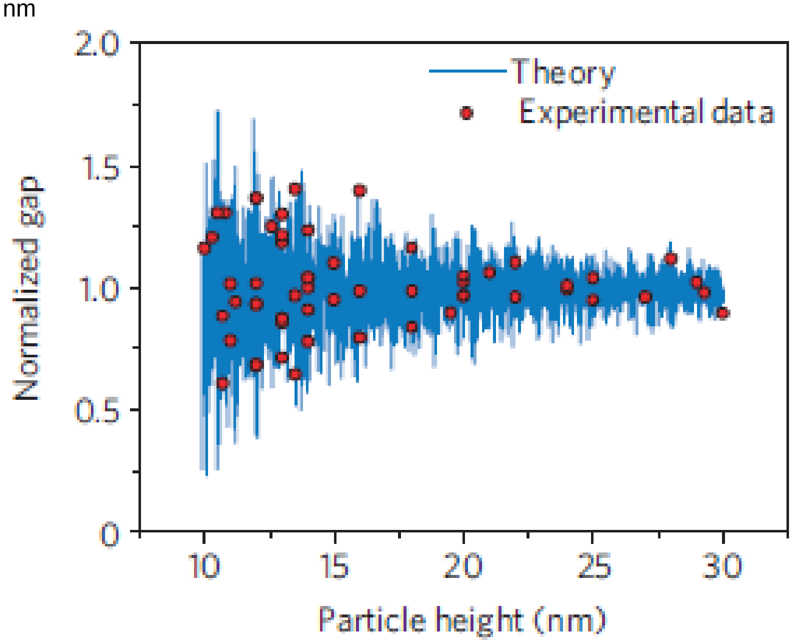
Higher T_c

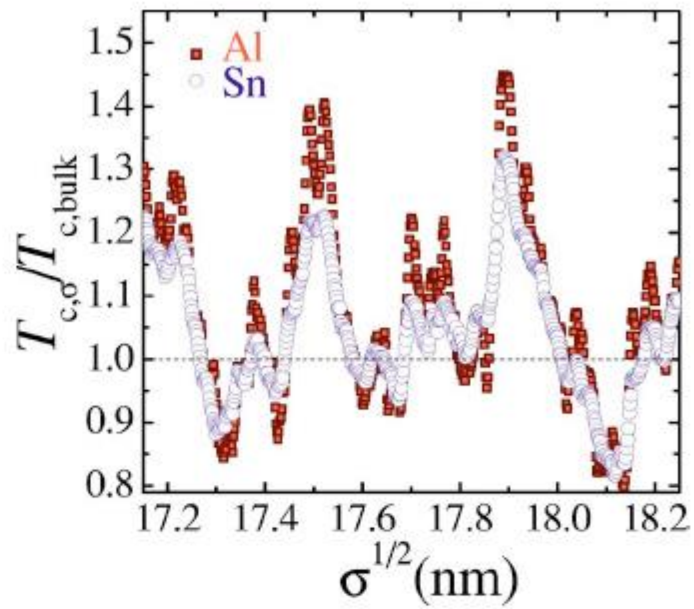
Xue et al.
Nature Communications 3, 931 (2012)

Single, Isolated grains









Peeters, Shanenko, 2007

S. Bose, AGG. et al., , Nature Materials 9, 550 (2010)

1d U < 0, T=0, Hubbard model

Korepin, 1989

$$\mathcal{H} = -\sum_{l\neq m,\sigma}^{L}\left(t_{lm}\hat{c}^{\dagger}_{l,\sigma}\hat{c}_{m,\sigma} + \mathrm{H.c.}\right) - \mu\sum_{l=1,\sigma}^{L}\left(\hat{n}_{l,\sigma}-\frac{1}{2}\right) - |U|\sum_{l=1}^{L}\left(\hat{n}_{l,\uparrow}-\frac{1}{2}\right)\left(\hat{n}_{l,\downarrow}-\frac{1}{2}\right)$$

SC

$$\langle \Psi^{+}_{n+j,\uparrow}\Psi^{+}_{n,\downarrow}\Psi_{i,\uparrow}\Psi_{1,\downarrow}\rangle \underset{n\to\infty}{\longrightarrow} \frac{1}{n^{\gamma}};$$

CDW

$$\langle\langle \Psi^{+}_{n,s}\Psi_{n,s}\Psi^{+}_{1,t}\Psi_{1,t}\rangle\rangle \underset{n\to\infty}{\longrightarrow} \cos(\pi D n)\frac{1}{n^{1/\gamma}}$$

D ~ 1

$$\gamma = 1 - \frac{1}{2\ln(\mathbb{C}/\delta)}$$

D << 1

$$\gamma = \frac{1}{2}\left(1 + \frac{D}{2}\sqrt{1 + \frac{1}{U^{2}}}\right)$$

SC wins over CDW

Coulomb and dissipation

1d Josephson chains

Bradley & Doniach

$C_0 \neq 0$

R=0

Dissipation?

Zaikin, Fazio

1d, U<0, continuous

Finite + Dissipation

Blatter

R=0

Infinite

Zaikin

$R=T^{\alpha}$

No long range order + Dissipation

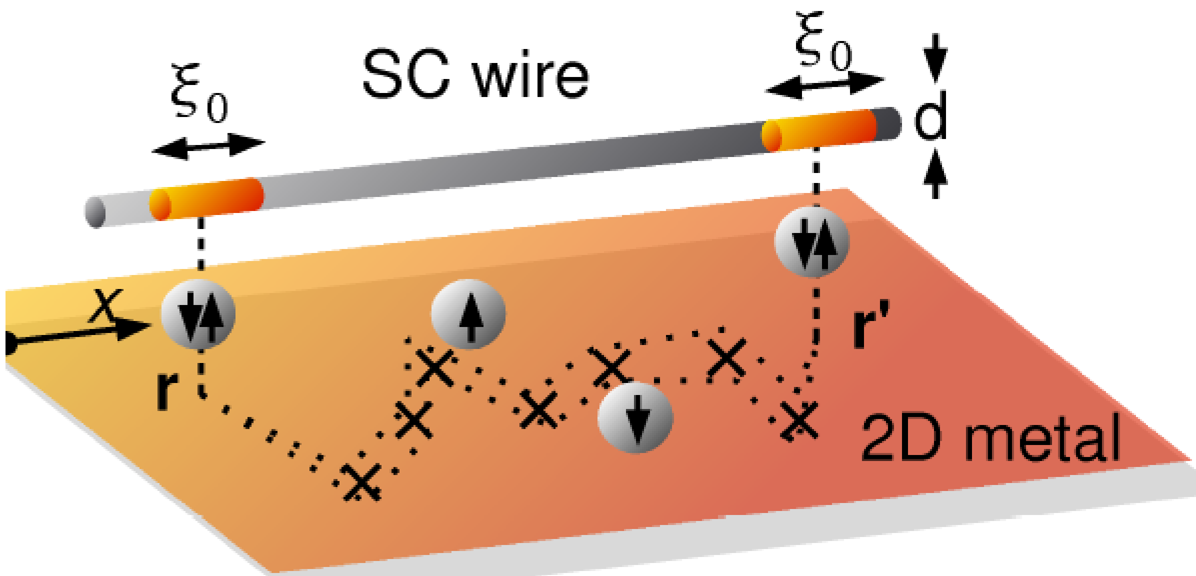
R=0

Fisher

Dissipation = Long range hopping in time

$$S_{diss}[\varphi] = \frac{\eta}{4\pi}\iint \left(\frac{\varphi(\tau)-\varphi(\tau')}{\tau-\tau'}\right)^{2} d\tau d\tau'$$

3/2



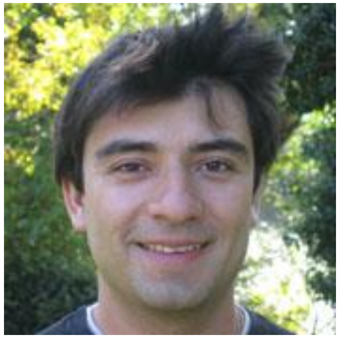
Lobos, Giamarchi, PRB 2009.

d = 1

Quasi long-range order

1d + Dissipation = Long range order

Giamarchi, Blatter, Zaikin, Fisher, Lobos..



A. Lobos
Maryland

Why not long-range order in 1d?

M. Tezuka
Kyoto

Phase coherence in 1d by power-law hopping

arXiv:1212.6779

1d Hubbard + power-law hopping

$$\mathcal{H} = -\sum_{l\neq m,\sigma}^{L}\left(t_{lm}\hat{c}^{\dagger}_{l,\sigma}\hat{c}_{m,\sigma} + \text{H.c.}\right) - \mu\sum_{l=1,\sigma}^{L}\left(\hat{n}_{l,\sigma} - \frac{1}{2}\right) - |U|\sum_{l=1}^{L}\left(\hat{n}_{l,\uparrow} - \frac{1}{2}\right)\left(\hat{n}_{l,\downarrow} - \frac{1}{2}\right)$$

$|U| \gg t$

$N/L \ll 1/2$

→ Slowest QLRO

$\propto 1/\sqrt{x}$

$$\mathcal{H}_{\text{eff}} = \frac{|U|\,L}{4} - \mu\sum_{l}(\hat{n}_l - 1) + \frac{4t^2}{|U|}\sum_{l\neq m}\left[\frac{(\hat{n}_l - 1)\,\hat{n}_m - \hat{\Delta}^{\dagger}_l\hat{\Delta}_m}{|l-m|^{2\alpha}} + \text{H.c.}\right]$$

$t_{lm} = t = 1 \qquad l = m \pm 1$

$t_{lm} = \lambda/|l-m|^{\alpha} \qquad l \neq m \pm 1$

Impossible review

Power-law hopping + U=0 Disorder → $$d_{eff} = \frac{2}{2\alpha - 1}$$

Mirlin, Fyodorov

Power-law hopping + U>0 Disorder → Thermalization without time scale

Rigol, Relano, AGG

Power-law α =1 + U>0 μ=1/2 Flux → Haldane Shastry

Gebhard and Ruckenstein

Bosonization

$$\rho(x) = \left[\rho_0 - \frac{\nabla\phi(x)}{\pi}\right] \sum_p e^{2ip(\pi\rho_0 x - \phi(x))}$$

$$\Delta(x) = \rho_0 e^{-i\theta(x)} \sum_p e^{2ip(\pi\rho_0 x - \phi(x))},$$

$$[\nabla\phi(x), \theta(y)] = i\pi\delta(x-y)$$

Phase

$$\langle\Delta(x)\rangle = \langle\hat{c}^\dagger_{x,\uparrow}\hat{c}^\dagger_{x,\downarrow}\rangle \propto \langle e^{-i\theta(x)}\rangle$$

Density

$$\delta\rho(x) \simeq -\nabla\phi(x)/\pi$$

$$S[\theta] = \frac{1}{\beta L}\sum_{\mathbf{q}}\left[\frac{K}{2\pi u}\omega_m^2 + \frac{uK}{2\pi}k^2\right]\theta_{-\mathbf{q}}\theta_{\mathbf{q}}$$

$$-\frac{\lambda u}{4a^3}\int d\tau \int_{|x-x'|>a} dx dx' \frac{1}{\left|\frac{x-x'}{a}\right|^{2\alpha}}\left[e^{i\theta(x)-i\theta(x')} + \text{H.c.}\right]$$

Self-consistent harmonic approximation (SCHA)

$$S_0 = \frac{1}{2\beta L} \sum_{\mathbf{q}} g_0^{-1}(\mathbf{q})\, \theta^*_{\mathbf{q}} \theta_{\mathbf{q}}$$

$$F_{\mathrm{var}} = F_0 + T \langle S - S_0 \rangle_0$$

$$g_0^{-1}(\mathbf{q}) = \frac{K}{\pi u}\omega_m^2 + \frac{uK}{\pi}k^2 + \frac{2\pi u \rho_0^2}{K a^{2-2\alpha}} \int_a^L dr \frac{1-\cos kr}{r^{2\alpha}} \times \exp\left[-\frac{1}{\beta L}\sum_{\mathbf{q}'} (1-\cos k' r)\, g_0(\mathbf{q}')\right]$$

RG?

$$g_0^{-1}(\mathbf{q}) = \frac{K}{\pi u}\omega_m^2 + \frac{uK}{\pi}k^2 + \eta\,|k|^{2\alpha-1}$$

$$\tilde{\eta} = \begin{cases} \left[4\pi\frac{\lambda\alpha}{K}\frac{\Gamma(-2\alpha)\sin(\pi\alpha)}{2^{\frac{1}{K(3-2\alpha)}}\tilde{k}_0^{\frac{1}{2K}}}\right]^{\frac{3-2\alpha}{3-2\alpha-1/2K}} & (\text{valid for } \tilde{\eta} \ll 1) \\ 4\pi\frac{\lambda\alpha}{K}\Gamma(-2\alpha)\sin(\pi\alpha) & (\text{valid for } \tilde{\eta} \gg \Gamma\left(\frac{3}{2}-\alpha\right)) \end{cases}$$

$\lambda \ll 1$

$$\alpha_c = \frac{3}{2} - \frac{1}{4K_{ren}}$$

$$K_{ren} > 1$$

$\lambda \gg 1$

$$\alpha_c = 3/2$$

$\alpha < \alpha_c$

Long range order

$\lambda \gg 1$

$\alpha > 3/2$	Quasi long-range order
$1/2 < \alpha < 3/2$	Long-range order

$$\langle \Delta(x_i) \rangle \sim \left\langle e^{i\theta(x_i)} \right\rangle \neq 0$$

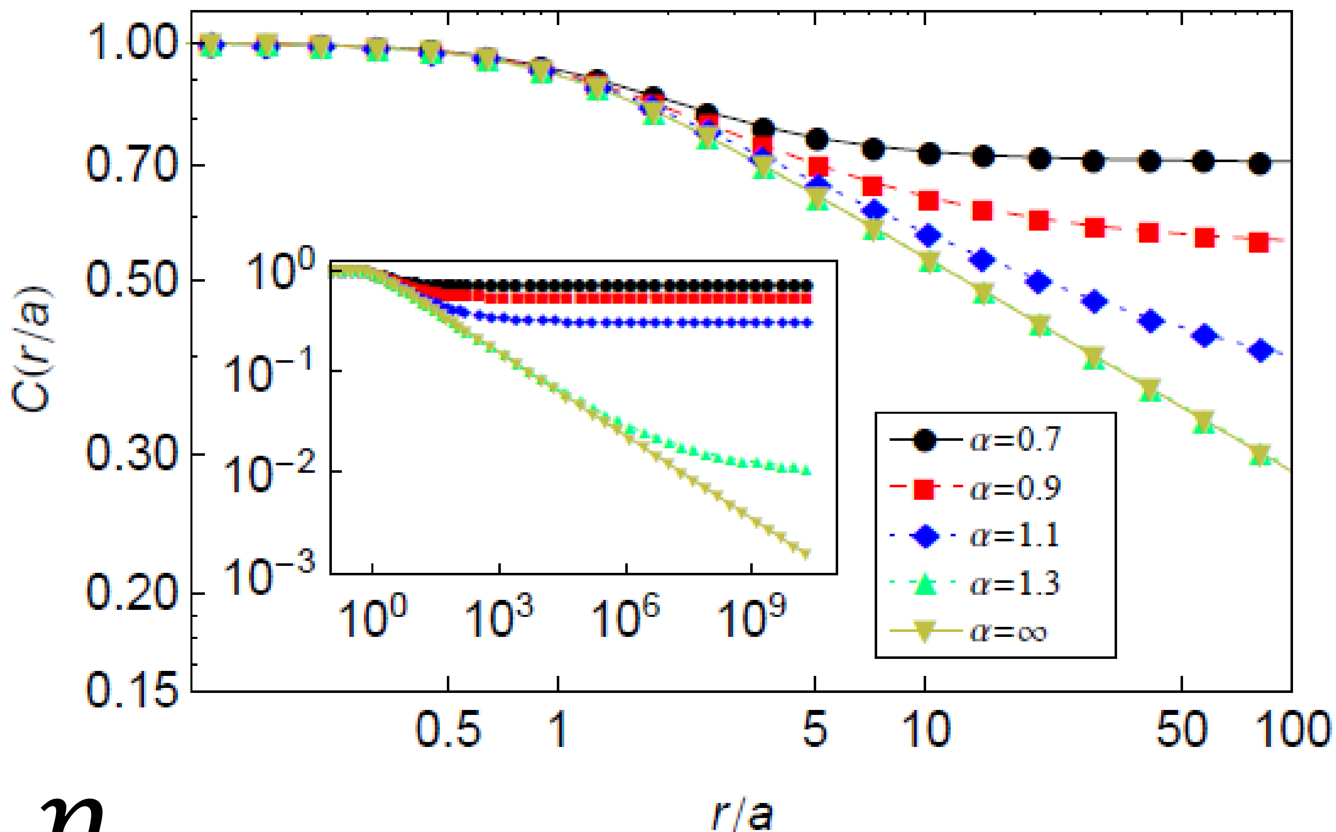
Phase slips suppressed

$d = 1$ BUT $d_{eff} = 2/(2\alpha - 1)$

$d_{eff} > 1$ Phase coherence

T > 0 still QLRO

Bosonization:Correlations



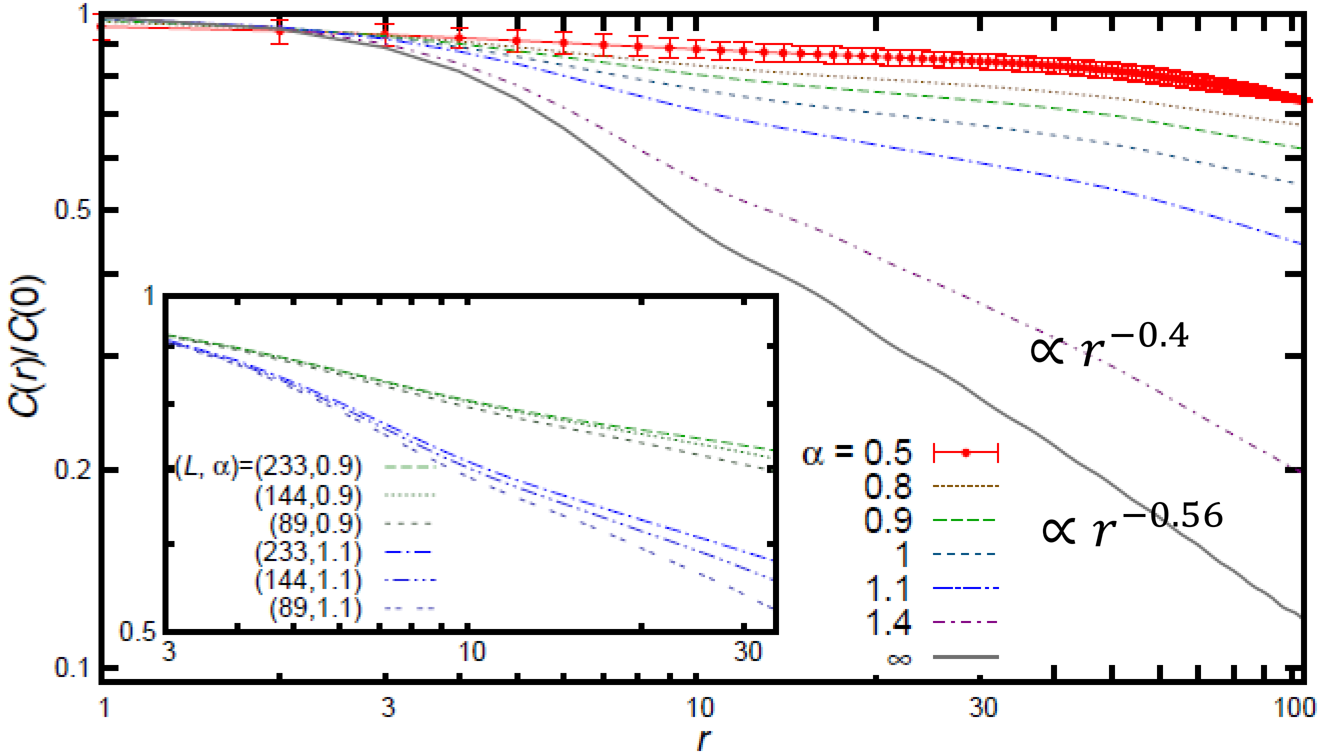
$$r \gg \eta$$

$$C(r) = \left\langle e^{i\theta(r)} e^{-i\theta(0)} \right\rangle_0$$

$$C(r) \approx e^{-G(0)} \left[1 + A/r^{3/2-\alpha} + \mathcal{O}\left(1/r^{3-2\alpha}\right)\right]$$

Numerical: DMRG

N/L ~ 0.15 U=-20 λ =1



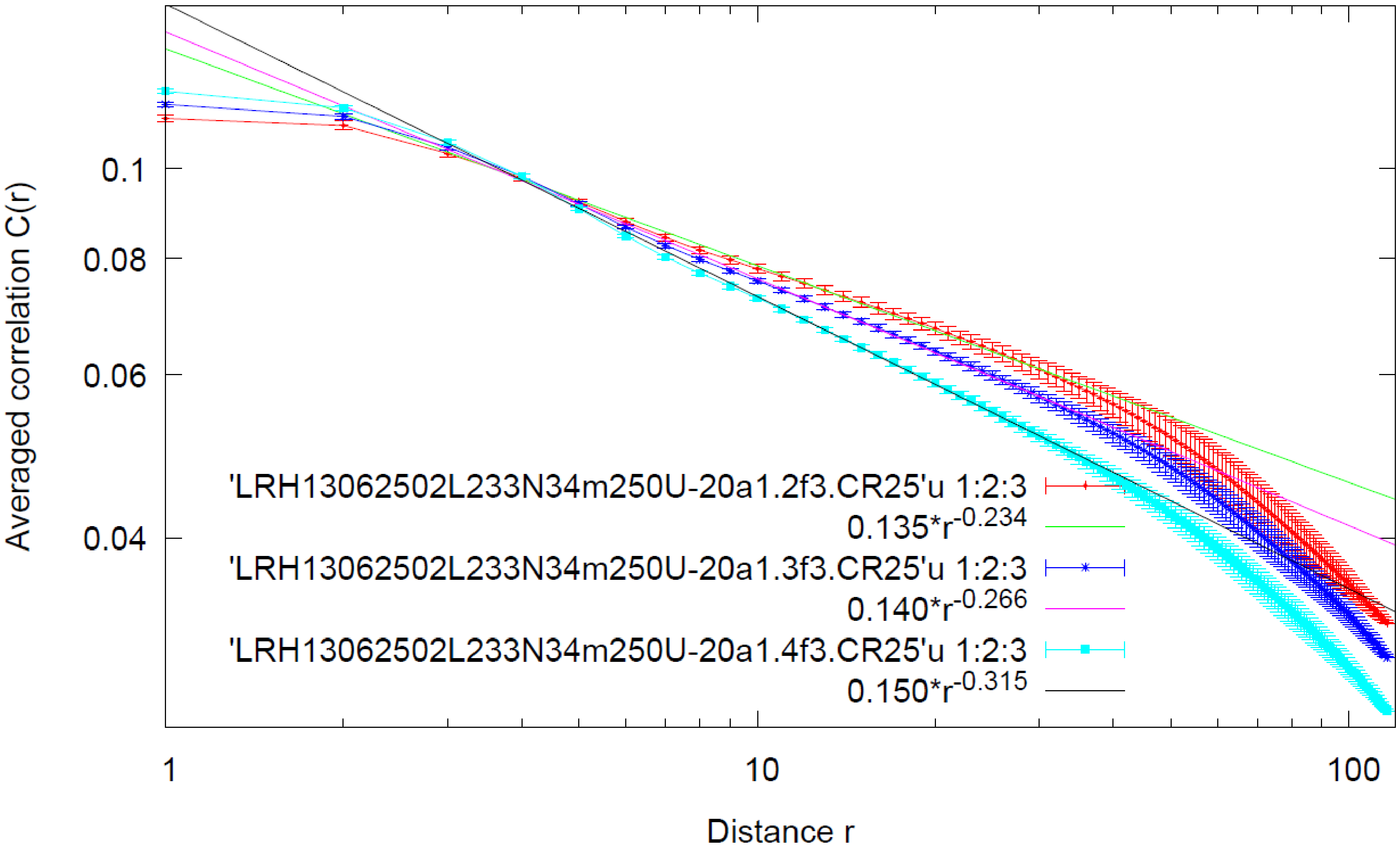
$$C(r) \equiv \frac{1}{L-2l_0-r} \sum_{l=l_0+1}^{L-l_0-r} \langle \hat{\Delta}_{l+r} \hat{\Delta}_l^\dagger \rangle$$

$$\hat{\Delta}_l^\dagger \equiv \hat{c}_{l,\uparrow}^\dagger \hat{c}_{l,\downarrow}^\dagger$$

U=-20

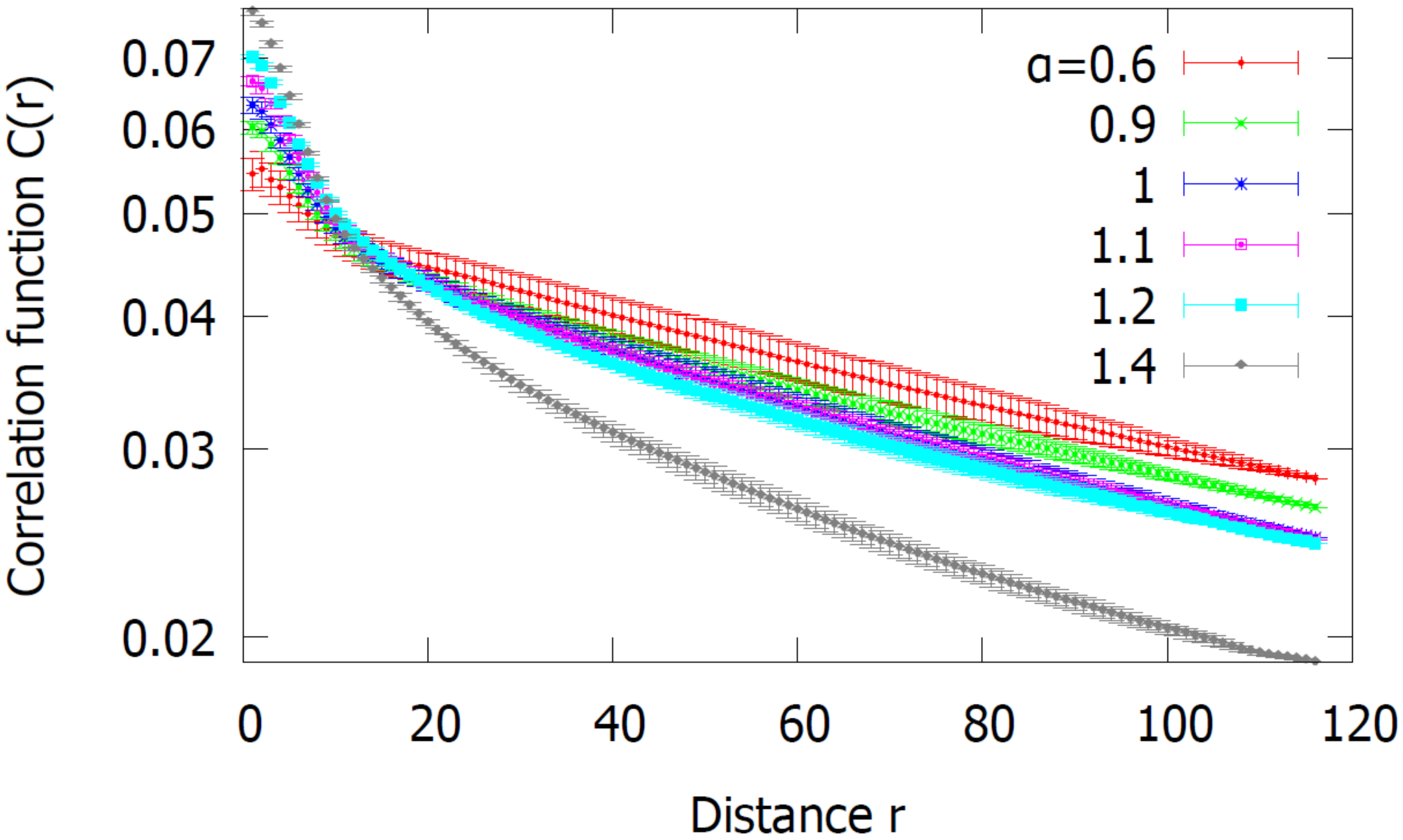
λ=3

N/L ~ 0.15



Averaged correlation C(r)
0.1
0.08
0.06
0.04
1
10
100
Distance r
'LRH13062502L233N34m250U-20a1.2f3.CR25'u 1:2:3
0.135*r^-0.234
'LRH13062502L233N34m250U-20a1.3f3.CR25'u 1:2:3
0.140*r^-0.266
'LRH13062502L233N34m250U-20a1.4f3.CR25'u 1:2:3
0.150*r^-0.315

λ=2.5   U=-20   N/L= 21/233



From SC to Quantum magnetism

$$\tilde{H}_{\text{eff}} = -2\mu \sum_{l} S_l^z + \sum_{l \neq m} \frac{8\,|t_{lm}|^2}{|U|} \left[S_l^z S_m^z \overset{+}{-} S_l^x S_m^x \overset{+}{-} S_l^y S_m^y - \frac{1}{4} \right]$$

Pseudo spin representation

$$\hat{\Delta}_l^\dagger \rightarrow \hat{S}_l^+$$

$$\hat{n}_l \rightarrow \hat{S}_l^z + 1/2$$

Real t_{ij}

XXZ Spin chain

Imaginary t_{ij}

Heisenberg chain

Spin Chains with long-range interactions

$$\mathcal{H} = \sum_i \left[\vec{S}_i \cdot \vec{S}_{i+1} - \lambda \sum_{j=2}^{\infty} (-1)^j \frac{\vec{S}_i \cdot \vec{S}_{i+j}}{j^\alpha} \right]$$

Does the phase diagram depends on λ?

No, Fisher, Phys. Rev. Lett. 29, 917 (1972).

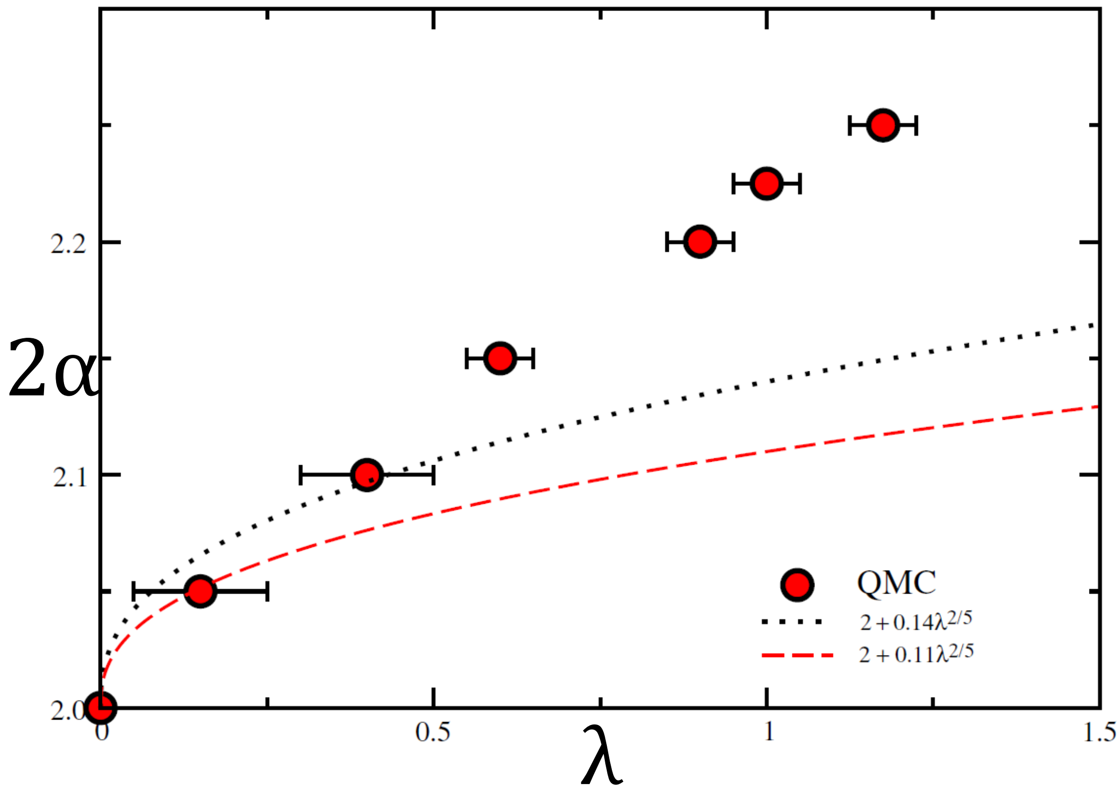
Yes, J. Sak, Phys. Rev. B 8, 281 (1973).

Summary: Blotte, Phys. Rev. Lett. 89, 025703 (2002)

Recent: Sperstad, Phys. Rev. B 85, 214302 (2012)

1d AFM

$$\mathcal{H} = \sum_i \left[\vec{S}_i \cdot \vec{S}_{i+1} - \lambda \sum_{j=2}^{\infty} (-1)^j \frac{\vec{S}_i \cdot \vec{S}_{i+j}}{j^\alpha} \right]$$



Differences between FM and AFM spin chains

$$C_{AFM}(r) \propto 1/r$$

$$C_{FM}(r) \propto 1/r^{1/2}$$

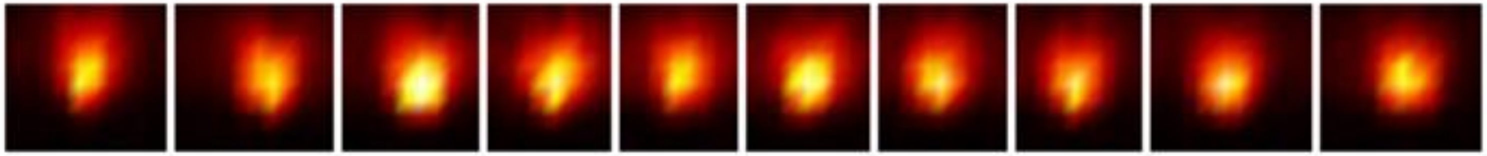
RG not reliable

Nicolas Laflorencie, I. Affleck, J. Stat. Mech. (2005) P12001

Variable-Range Interactions in Trapped Ion Quantum Simulators

Islam, Monroe, Science 340, 583 (2013) Bollinger, Britton, Nature 484, 489 (2012)

$^{171}Yb^+$



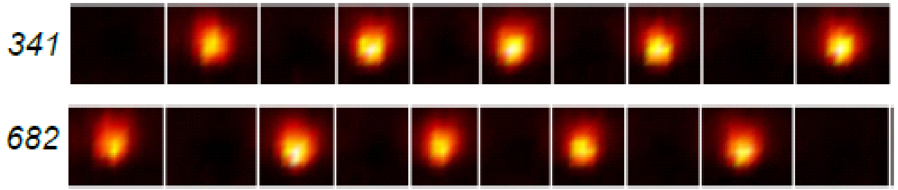
Raman transitions

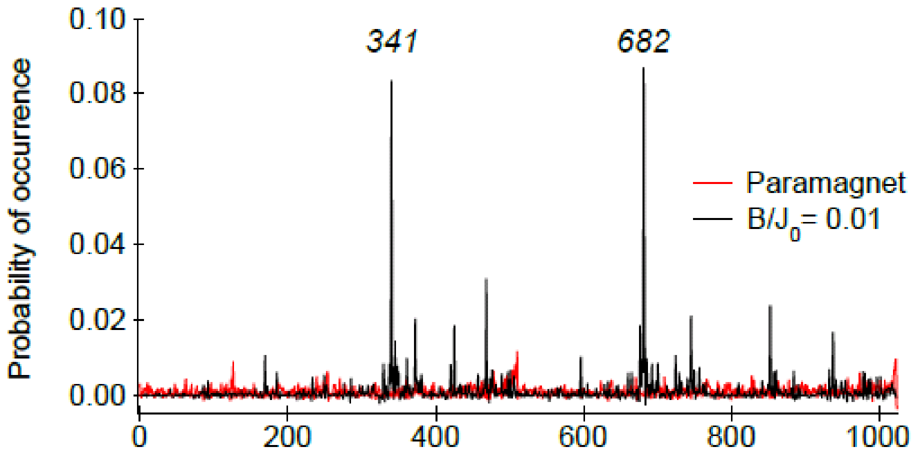
π/2-shift

Dipole forces

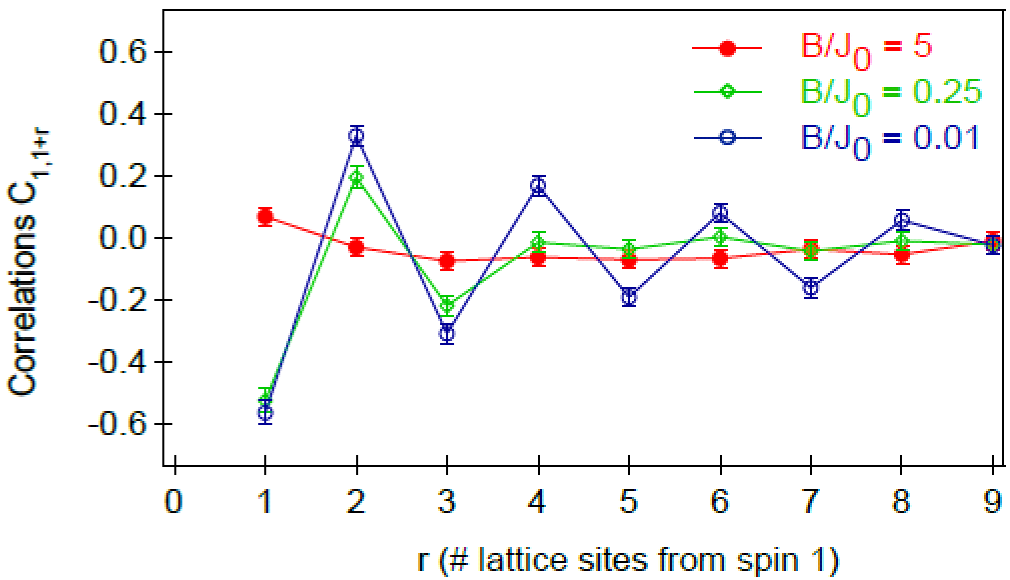
$$H = \sum_{j<i} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} - B\sum_i \sigma_y^{(i)} \qquad J_{ij} \approx \frac{J_0}{|i-j|^\alpha}$$

$$0 < \alpha < 3$$





$$C_{1,1+r} = \langle \sigma_x^{(1)} \sigma_x^{(1+r)} \rangle - \langle \sigma_x^{(1)} \rangle \langle \sigma_x^{(1+r)} \rangle$$



Ferromagnetic Transitions

Frustration

Spin Liquids

Quantum Magnetism

Summary

Different tricks are available to restore LRO in low dimensional superconductors

Different tricks are available to enhance superconductivity in low dimensions

Let’s combine this so that:

Substantial enhancement of T_c

THANKS!