Universal quantum constraints on the butterfly effect

Berenstein, A. Garcia-Garcia (Cambridge U.), arXiv:1510.08870

David Berenstein
UC Santa Barbara

The out of equilibrium birth of a superfluid

Cherler, AGG, Liu, Phys. Rev. X 5, 021015 (2015)

Hong Liu
MIT

Paul Chesler
Harvard

Butterfly effect

Classical chaos

Hadamard 1898

Alexandr Lyapunov 1892

$$\|\delta x(t)\| = e^{\lambda t}\|\delta x(0)\|$$

$$\lambda > 0$$

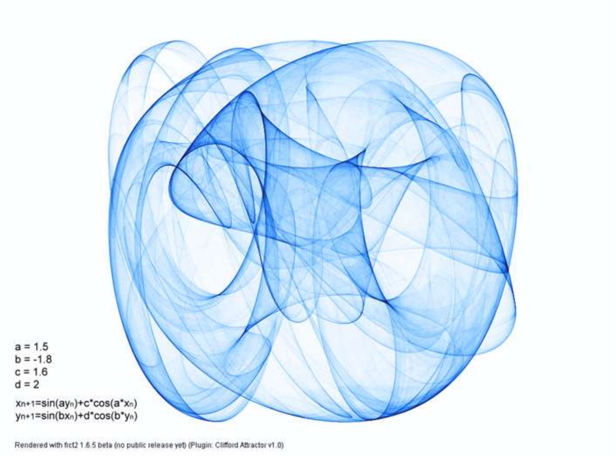
$$h_{KS} > 0$$

Pesin theorem

Difficult to compute!

Lorenz 60's

Meteorology



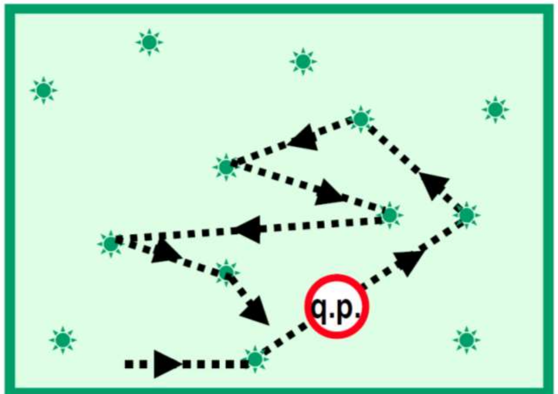
Quantum chaos?

Role of classical chaos in the $\hbar \to 0$ limit

Quantum butterfly effect?

Disordered system

Larkin, Ovchinnikov,
Soviet Physics JETP 28, 1200 (1969)



Altshuler, Lancaster lectures

$\langle p_z(t)p_z(0)\rangle \propto e^{-t/\tau}$

τ Relaxation time

$$\langle [p_z(t), p_z(0)]^2\rangle \approx \hbar^2 \langle \{p_z(t), p_z(0)\}^2\rangle \propto \hbar^2 \exp(\lambda t)$$

$\tau \ll t < t_E \sim \log \hbar^{-1}/\lambda$ Chaotic

$t_E \propto \hbar^\alpha \ \alpha < 0$ Integrable

Quantum chaos?

CONDITION OF STOCHASTICITY IN QUANTUM NONLINEAR SYSTEMS

Physica 91A 450 (1978)

G.P. BERMAN and G.M. ZASLAVSKY

Kirensky Institute of Physics, Siberian Department of the Academy of Sciences, Krasnoyarsk, 660036, USSR

$$F(t) = F_0 \sum_{n=-\infty}^{\infty} \delta(t - nT) \qquad [a, a^\dagger] = \hbar$$

$$H = H_0 + \epsilon V;$$

$$H_0 = \omega a^\dagger a + \mu (a^\dagger a)^2; \qquad V = F(t)(a^\dagger + a); \qquad \mu > 0,$$

Mapping of operators in Heisenberg picture

$$(a_{n+1}, a^\dagger_{n+1}) = \hat{T}(a_n, a^\dagger_n)$$

Projection on coherent states = classical map + quantum corrections

$$a_0|\alpha_0\rangle = \alpha_0|\alpha_0\rangle$$

$$\alpha_n \equiv \langle a_n \rangle = a_n^{(N)}(\alpha_0^*, \alpha_0),$$
$$\alpha_n^* \equiv \langle a_n^\dagger \rangle = a_n^{\dagger(N)}(\alpha_0^*, \alpha_0)$$

$$(\alpha_{n+1}, \alpha^*_{n+1}) = \hat{\mathcal{T}}(\alpha_n, \alpha^*_n)$$

$$I_n = |\alpha_n|^2; \qquad \varphi_n = \frac{1}{2\,\mathrm{i}} \ln\left(\frac{\alpha_n}{\alpha_n^*}\right)$$

$$\begin{aligned} I_{n+1} = {} & I_n - 2\epsilon F_0 I_n^{1/2} \sin \varphi_n + \epsilon^2 F_0^2 + 4\hbar\beta_n \mu T I_n (\sin \varphi_n - \cos \varphi_n) \\ & - 4\hbar\beta_n T \mu \epsilon F_0 I_n^{1/2} (2 + \cos 2\varphi_n + \sin 2\varphi_n - \cos \varphi_n), \end{aligned}$$

$$\begin{aligned} \varphi_{n+1} = {} & \varphi_n - (\omega + \mu\hbar)T - \epsilon F_0 I_n^{-1/2} \cos \varphi_n - \tfrac{1}{2}\epsilon^2 F_0^2 I_n^{-1} \sin 2\varphi_n \\ & -2\mu T(I_n - 2\epsilon F_0 I_n^{1/2} \sin \varphi_n + \epsilon^2 F_0^2) - 2\hbar\mu T\beta_n \\ & -4\hbar\mu T\beta_n \epsilon^2 F_0^2 I_n^{-1}(1 + \sin^2 \varphi_n), \end{aligned}$$

$lnK = Lyapunov$

$$\beta_n \equiv \frac{1}{4} \frac{I_n}{I_0} \left(\frac{\partial \varphi_n}{\partial \varphi_0}\right)^2$$

$\tau \ll t < t_E \sim \log \hbar^{-1} / lnK$

$$\beta_n = \tfrac{1}{4} \exp n \left[2 \ln \bar{K} + \kappa + \frac{\langle\langle \Delta I \rangle\rangle}{I} \right]$$

Quantum butterfly effect

Why is quantum chaos relevant?

Quantum classical transition

Quantum Information

Prepare a classically chaotic system

Couple it to a thermal reservoir

Compute the growth of the entanglement entropy by integrating the reservoir

?

Zurek-Paz conjecture

Phys. Rev. Lett. 72, 2508 (1994)
Phys. Rev. Lett. 70, 1187 (1993)

Berenstein, Asplund, Annals of Physics 366 (2016) 113

Oscillators + thermal bath

$$S = -Tr[\rho_A \log \rho_A] \qquad \rho_A = Tr_B \rho_{AB}$$

$$S \approx h_{KS} t = \Sigma \lambda_i t \qquad t < t_E$$

Decohorence is controlled by classical chaos not the reservoir!

Numerical evidence?

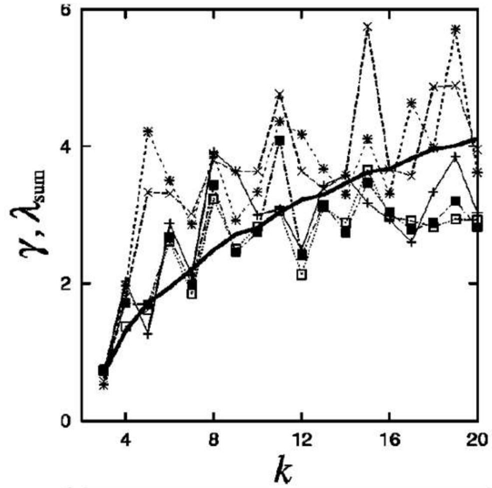
Yes, but…

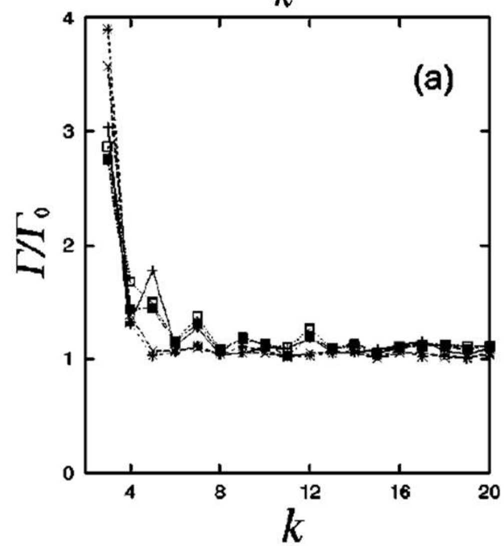
Coupled kicked tops

Phys. Rev. E 67 (2003) 066201

$$H(t) = H_1(t) + H_2(t) + H_\epsilon(t)$$

$$\begin{aligned}
H_1(t) &= \frac{k_1}{2j} J_{z_1}^2 \sum_n \delta(t-n) + \frac{\pi}{2} J_{y_1}, \\
H_2(t) &= \frac{k_2}{2j} J_{z_2}^2 \sum_n \delta(t-n) + \frac{\pi}{2} J_{y_2}, \\
H_\epsilon(t) &= \frac{\epsilon}{j} J_{z_1} J_{z_2} \sum_n \delta(t-n), \\
S_{\mathrm{vN}}(t) &= -\mathrm{Tr}_1\{\rho^{(1)}(t) \ln \rho^{(1)}(t)\}, \\
S_{\mathrm{lin}}(t) &= 1 - \mathrm{Tr}_1\{\rho^{(1)}(t)^2\},
\end{aligned}$$





Not always

$$\Gamma_0 = S_0 D_0$$

$$D_0 \sim 1, S_0 = \epsilon^2 j^2$$

$$\Gamma \equiv \left. \frac{dS_{\mathrm{lin}}^{\mathrm{PT}}(t)}{dt} \right|_{t \gg 1/\gamma} \simeq \Gamma_0 \coth(\gamma/2)$$

$$S_{\mathrm{lin}}^{\mathrm{PT}}(t) \simeq S_0 D_0 \left[\coth(\gamma/2)\, t - \frac{1 - e^{-\gamma t}}{\sinh \gamma - 1} \right]$$

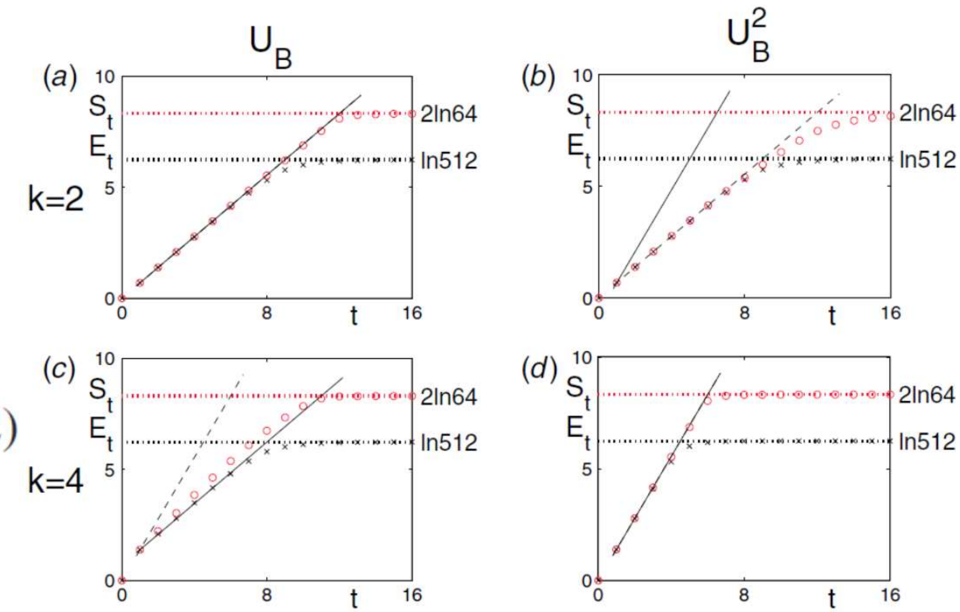
Noisy environment

Quantum Baker map

$$(q, p) \to T_B(\gamma) = (2q - [2q], (p + [2q])/2)$$

$$U_B = (\mathcal{F}_d)^{-1} \cdot \begin{pmatrix} \mathcal{F}_{d/2} & 0 \\ 0 & \mathcal{F}_{d/2} \end{pmatrix}$$

$$\sum_j [\mathcal{F}_d]_{kj} e_j = \sum_j \frac{1}{\sqrt{d}} \mathrm{e}^{-2\pi i k j/d} e_j$$



Alicki, 2003

$$\rho \mapsto \Phi_{\mathbf{X}}(\rho) = \sum_{j=1}^{\kappa} X_j \rho X_j^\dagger$$

$$\sum_{j=1}^{k} X_j^\dagger X_j = \mathbb{1}$$

$$S_t[\mathbf{X}, U] \leqslant \min\{t \ln k, d\}$$

$$h_{KS} > \ln k$$

Any environment may limit the growth of the entanglement entropy!

Why should you care at all about this?

Fast Scramblers

Sekino, Susskind,JHEP 0810:065,2008

P. Hayden, J. Preskill, JHEP 0709 (2007) 120

t_E?

1. Most rapid scramblers take a time logarithmic in N
2. Matrix quantum mechanics saturate the bound
3. Black holes are the fastest scramblers in nature

(Quantum) black hole physics | AdS/CFT | Strongly coupled (quantum) QFT

Why?

Rindler!

All thermal horizon are locally isomorphic to Rindler geometry

Rest charge at z_c

Stretched horizon $\rho = l_p$

$$\begin{aligned} \rho^2 &= z^2 - t^2 \\ z &= \rho \cosh \omega \\ t &= \rho \sinh \omega. \end{aligned}$$

$$ds^2 = -\rho^2 d\omega^2 + d\rho^2 + dx_\perp^2$$

$$E_\rho = E_z = \frac{e(z - z_c)}{[(z - z_c)^2 + x_\perp^2]^{\frac{3}{2}}} = \frac{e(\rho \cosh \omega - z_c)}{[(\rho \cosh \omega - z_c)^2 + x_\perp^2]^{\frac{3}{2}}}$$

$$\omega \gg 1$$

$$\sigma = \frac{1}{4\pi\rho} E_\rho|_{\rho_{SH}} = \frac{e}{4\pi \ell_p} \frac{\ell_p e^\omega}{[(\ell_p e^\omega)^2 + x_\perp^2]^{\frac{3}{2}}}$$

Spread of charge density $\quad \Delta x \sim l_p e^{\omega}$

Like quantum chaos!

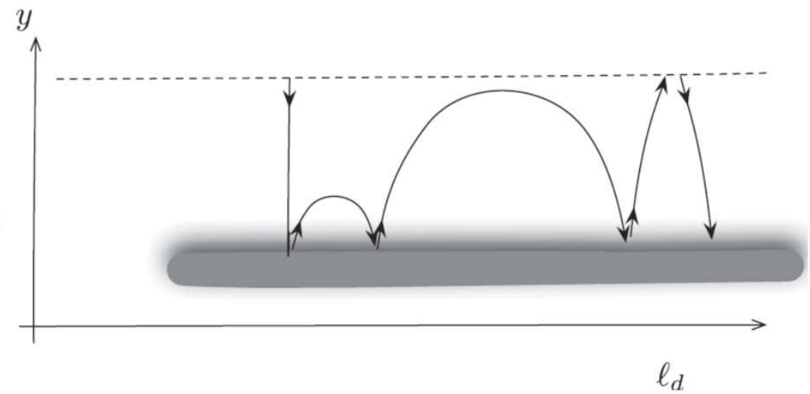
Scrambling time black hole $\quad \omega_* \sim \log R_s / l_p \qquad t_* \sim \beta \log S$

Typical Scrambling time $\quad t_* \sim \beta S^{2/d}$

Black hole are fast(est) scramblers

Dual interpretation of scrambling

Barbon, Magan, PRD 84, 106012 (2011) Chaotic fast scrambling at black holes



$N \to \infty$ $\quad \langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim e^{-t/\tau_\beta}$

Only Quasinormal modes

Finite N Probe in a hyperbolic “billiard”

M.C.Gutzwiller Chaos in Classical and Quantum Mechanics Springer-Verlag, New York, 1990

Hard chaos

$$ds^2_{\rm op} \approx -dt^2 + dz^2 + e^{4\pi T(z-z_\beta)} d\ell^2 \qquad ds^2_{\rm op} \approx -dt^2 + \left(\frac{\beta}{2\pi}\right)^2 ds^2_{\mathbf{H}^{d+1}}$$

$$\tau_* \sim \beta \log\left(\frac{S}{n_{\rm cell}}\right) = \beta \log(S_{\rm cell})$$

$$S_{cell} \sim N_{eff} \sim N^2 \;\; CFT$$

Only for small systems $1/\beta$

Black holes and the butterfly effect

Shenker, Stanford, arXiv:1306.0622

Sensitivity to initial conditions in the dual field theory

Holography calculation

2+1 BTZ

Mild pertubation $E_p \sim \frac{E\ell}{R} e^{Rt_w/\ell^2}$ BTZ shock waves

Mutual information $I = S_A + S_B - S_{A\cup B}$

$$I(A;B) = \frac{\ell}{G_N}\left[\log\sinh\frac{\pi\phi\ell}{\beta} - \log\left(1 + \frac{E\beta}{4S} e^{2\pi t_w/\beta}\right)\right]$$

$I \sim 0$

$$t_*(\phi) = \frac{\phi\ell}{2} + \frac{\beta}{2\pi}\log\frac{2S}{\beta E} \qquad \beta E \sim 1, S \sim N^2 \qquad t_* = \frac{\beta}{2\pi}\log S$$

A bound on chaos

Juan Maldacena[1], Stephen H. Shenker[2] and Douglas Stanford[1]

$$y^4 = \frac{1}{Z} e^{-\beta H} \qquad F(t) = \mathrm{tr}[yVyW(t)yVyW(t)]$$

$$t_* = \frac{\beta}{2\pi} \log N^2 \quad F_d \equiv \mathrm{tr}[y^2 V y^2 V]\mathrm{tr}[y^2 W(t) y^2 W(t)]$$

$$t_d \ll t < t_* \qquad F_d - F(t) = \epsilon \exp \lambda_L t + \cdots \qquad \epsilon \sim 1/N^2$$

Large N CFT $\quad F(t) = f_0 - \frac{f_1}{N^2} \exp \frac{2\pi}{\beta} t + \mathcal{O}(N^{-4})$

$$\lambda_L \leq \frac{2\pi}{\beta} = 2\pi T$$

Not in agreement with the Zurek-Paz conjecture

Lyapunov exponent is a classical quantity

Exponential growth has to do with classical chaos

?

Berenstein,AGG arXiv:1510.08870

How is this related to quantum information?

Are there universal bounds on Lyapunov exponents and the semiclassical growth of the EE?

How universal?

Environment

Quantumness

Quantumness: Size of Hilbert space limits growth of EE

$$\Delta x_n \Delta x_0 \geq |[\hat{x}_n, \hat{x}_0]|/2$$

$$\Delta x_1 < \Delta x_{max} \simeq A \qquad \Delta x_0 \approx \sqrt{\hbar}$$

$$A\sqrt{\hbar} > \Delta x_0 \Delta x_1 \geq |[\hat{x}_1, \hat{x}_0]|/2 \approx \hbar e^{\kappa_+ \tau}$$

Discrete time $\quad N \sim \Delta x \Delta p/\hbar \qquad \kappa_+ < B \log(\hbar^{-1})$

$$\kappa_+ = \lambda < \mathrm{B} \log N$$

$$\tau \ll t \leq t_E \sim \log \hbar^{-1}/\lambda$$

$$\tau \ll t \leq t_E \sim \log \hbar^{-1}/\lambda$$

$$\boxed{S \sim \lambda t < tB \log N}$$

Classical Lyapunov exponents larger than log N do not enter in semiclassical expressions

Quantum information

S. Bravyi, Phys. Rev. A 76, 052319 (2007).
F. Verstraete et al.,Phys. Rev. Lett. 111, 170501 (2013).

$$\frac{\Delta S}{\Delta n} < \mathrm{A} \log d$$

Bipartite systems

No semiclassical interpretation

Arnold cat map

$$U^N = V^N = 1$$

$$V \simeq \exp(2\pi i\hat{p}) \quad U \simeq \exp(2\pi i\hat{x})$$

$$\begin{pmatrix} x \\ p \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix} = M \begin{pmatrix} x \\ p \end{pmatrix}$$

x, p *periodic*

$$\hbar \sim 1/2\pi N$$

$$a = 2, b = c = d = 1$$

$$UV = \exp(2\pi i/N)VU$$

$$U_n U - U U_n = \left(1 - \exp\left[\frac{2\pi i}{N}(M^n)_{12}\right]\right) U_n U$$

$$\Delta x_n \Delta x_0 \geq |[\hat{x}_n, \hat{x}_0]|/2$$

$$\Delta U_1 \frac{1}{\sqrt{N}} \geq \frac{1}{N} \exp(\lambda_+)$$

$$\lambda_+ \leq \log(\sqrt{N})$$

1d lattice of cat maps

time step = effective light-crossing time per site

$$m << k - m$$

$$\tilde{M}_{nn} = \begin{pmatrix} \ddots & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ & & & & & \ddots \end{pmatrix} \qquad \tilde{M}_{\Gamma} = \begin{pmatrix} \ddots & & \\ & M & \\ & & \ddots \end{pmatrix}$$

$$S \sim \sum \log \beta_i n \qquad M_{tot} = \tilde{M}_{nn}.\tilde{M}_{\Gamma} \qquad \beta_i \sim e^{k_{max}}$$

$$\frac{\Delta S}{\Delta n} \approx 2mk_{max} < \log N \quad \propto V$$

Entanglement is a local phenomenon

Also $S \propto \alpha n$

but

Thermalization of Strongly Coupled Field Theories
deBoer, Vakkuri, et al., Phys. Rev. Lett. 106, 191601(2011)

Only for $t \leq t_T$

Entanglement Tsunami

Liu, Suh, Phys. Rev. Lett. 112, 011601 (2014)

$S \propto A$ (not V)

Bound induced by the environment

Single particle coupled to a thermal bath

Aslangul et al., Journal of Statistical Physics (1985) 40, 167

$$H = \frac{P^2}{2M} + \sum_n M\Omega_n^2 X x_n + \sum_n \frac{p_n^2}{2m_n} + \sum_n \frac{1}{2} m_n \omega_n^2 x_n^2 + \sum_n \frac{1}{2} \frac{M^2\Omega_n^4}{m_n\omega_n^2} X^2$$

$$\ddot{X}(t) = -A(t) - \int_{t_0}^{t} dt'\, K(t-t')\, \dot{X}(t') \qquad K(t) = \frac{\gamma}{\tau_R} e^{-\gamma t}\Theta(t)$$

Random force correlation

$$\Phi_T(t) = \tfrac{1}{2}\langle A(t+t')\, A(t') + A(t')\, A(t+t')\rangle$$

$$\Phi_T(t) \simeq \frac{\hbar\gamma^2}{2\pi M\tau_R} \times \begin{cases} -2(C+\ln\gamma t), & 0 < t \lesssim \gamma^{-1} \\ -2/(\gamma t)^2, & \gamma^{-1} \lesssim t \lesssim \tau \\ -2(\gamma\tau)^{-2} e^{-t/\tau}, & \tau \lesssim t \end{cases} \qquad \tau = (2\pi)^{-1}\frac{\hbar}{k_B T}$$

$$\lambda \gg 1/\tau \qquad\qquad \lambda \ll 1/\tau$$

$$\langle [p_z(t), p_z(0)]^2\rangle \propto e^{t/\tau} \qquad\qquad \langle [p_z(t), p_z(0)]^2\rangle \propto e^{\lambda t}$$

$$S \sim t/\tau \qquad\qquad S \sim \lambda t$$

QM Noise limits the butterfly effect

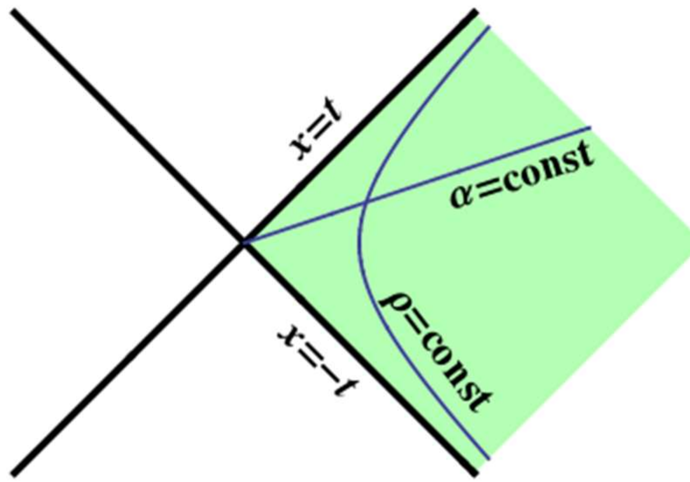
Maximum (?) Rate of information loss

Membrane paradigm

$$\Delta x(0)\Delta p(0) \approx \hbar$$

$$G \propto 1/N^2 \qquad p \sim e^{t/4MG}$$

$$\Delta x^2 \propto p \sim Ge^{t/4MG}$$



Rindler geometry

$$S \sim \log(\Delta X \Delta P) \sim \frac{t}{4MG} \sim 2\pi k_B T t/\hbar$$

Causality constraints

+

Quantum Noise

$$p \leq e^{t/4MG}$$

$$\boxed{S \sim t/\tau}$$

$$\boxed{\tau \geq \hbar/2\pi k_B T}$$

ρ_0 Stretched Horizon

$$X^i = 0, t = 0, z = \rho_0$$

Forward Light Cone

$$R'^2 = x^i x^i \qquad t^2 = (z - \rho_0^2) + R'^2$$

Intersection light cone
with stretched horizon

$$R'^2 = 2z\rho_0 - 2\rho_0{}^2$$

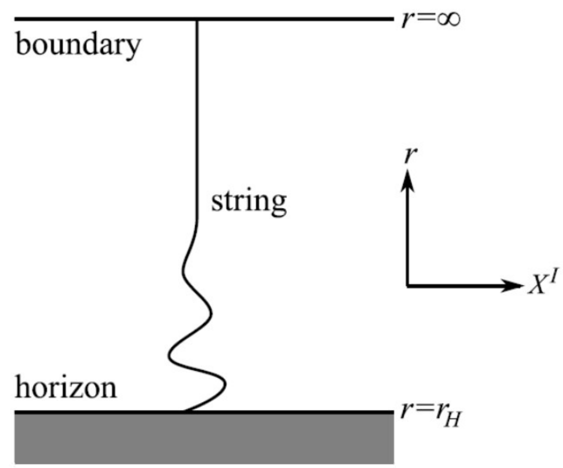
Large times

$$R'^2 \approx \rho_0{}^2 e^{t/4MG}$$

QM induces entanglement
but also limits its growth

Brownian motion in AdS/CFT

deBoer, Hubeny,JHEP 0907:094,2009



$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d^2x\sqrt{-\det\gamma_{\mu\nu}}$$

$$\approx -\frac{1}{4\pi\alpha'}\int d^2x\sqrt{-g(x)}\,g^{\mu\nu}(x)\,G_{IJ}(x)\,\frac{\partial X^I}{\partial x^\mu}\frac{\partial X^J}{\partial x^\nu} \equiv S^{(2)}_{\rm NG}$$

$$\left[-\partial_t^2 + \frac{r^2-r_H^2}{\ell^4\,r^2}\,\partial_r\left(r^2\,(r^2-r_H^2)\,\partial_r\right)\right]X(t,r) = 0$$

Hawking radiation

$$X(t,r) = \sum_{\omega>0}\left[a_\omega u_\omega(t,\rho) + a_\omega^\dagger u_\omega(t,\rho)^*\right]$$

$$\langle a_\omega^\dagger\, a_{\omega'}\rangle = {\rm Tr}\left(\rho_0\, a_\omega^\dagger\, a_{\omega'}\right) = \frac{\delta_{\omega\,\omega'}}{e^{\beta\omega}-1}$$

$$x(t) \equiv X(t,\rho_c) = \sum_{\omega>0}\sqrt{\frac{2\alpha'\,\beta}{\ell^2\,\omega\,\log(1/\epsilon)}}\left[\frac{1-i\nu}{1-i\rho_c\nu}\left(\frac{\rho_c-1}{\rho_c+1}\right)^{i\nu/2} e^{-i\omega t}\,a_\omega + {\rm h.c.}\right]$$

$$\dot{p}(t) = -\int_{-\infty}^{t} dt'\, \gamma(t-t')\, p(t') + R(t)$$

$$\kappa^{\mathrm{n}}(t) = \langle : R(t) R(0) : \rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} I_R^{\mathrm{n}}(\omega)\, e^{-i\omega t}$$

$$\kappa^{\mathrm{n}}(\omega) = I_R^{\mathrm{n}}(\omega) = \frac{I_p^{\mathrm{n}}(\omega)}{|\mu(\omega)|^2} = \frac{4\pi\ell^2}{\alpha'\beta^3} \frac{1+\nu^2}{1+\rho_c^2\nu^2} \frac{\beta|\omega|}{e^{\beta|\omega|}-1}$$

$$\kappa^{\mathrm{n}}(t) \approx \frac{2\ell^2}{\alpha'\beta^4} h_1(t,\beta) = \frac{2\ell^2}{\alpha'\beta^4}\left[\left(\frac{\beta}{t}\right)^2 - \frac{\pi^2}{\sinh^2(\pi t/\beta)}\right]$$

$$\langle [p(t), p(0)]^2 \rangle \propto \hbar^2 \exp(t/\tau)$$

$$S \sim t/\tau$$

$$\tau = \hbar/2\pi k_B T$$

?

Quantum mechanics induces entanglement
but also limits its growth rate

Environment modifies the semiclassical analysis of
the entanglement growth rate

Is the growth rate bound universal beyond the
semiclassical limit?

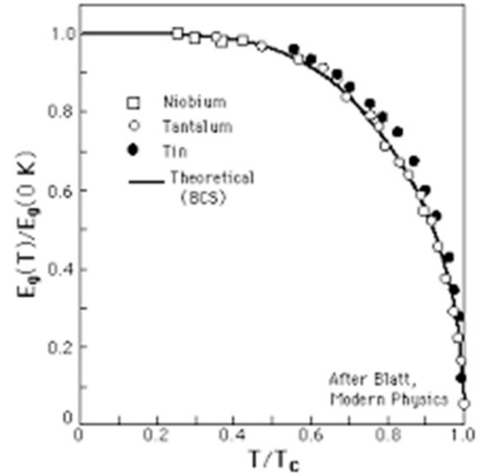
To what extent is the environment effect
universal, extremal black hole?

Can holography say something about it?

Not easy!

The out of equilibrium birth of a superfluid

Phys. Rev. X 5, 021015 (2015)



Hong Liu
MIT

Paul Chesler
Harvard

$$\xi_{eq} = \xi_0 |\epsilon|^{-\nu}$$

$$\tau_{eq} = \tau_0 |\epsilon|^{-\nu z}$$

Unbroken Phase

Broken phase

T(t) $\langle \psi \rangle = 0$

T_c $\langle \psi \rangle \neq 0$

$$\langle \psi \rangle = \Delta(x,t) e^{i\theta(x,t)}\ ?$$

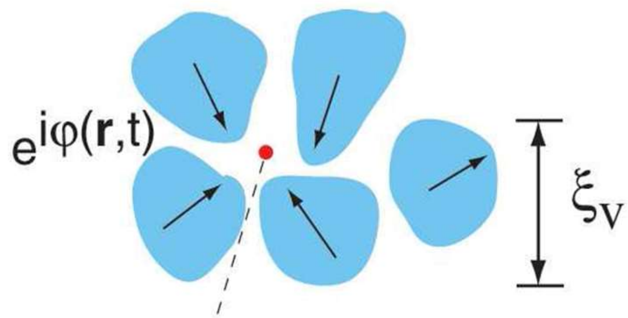
Kibble

J. Phys. A: Math. Gen. 9: 1387. (1976)

Causality

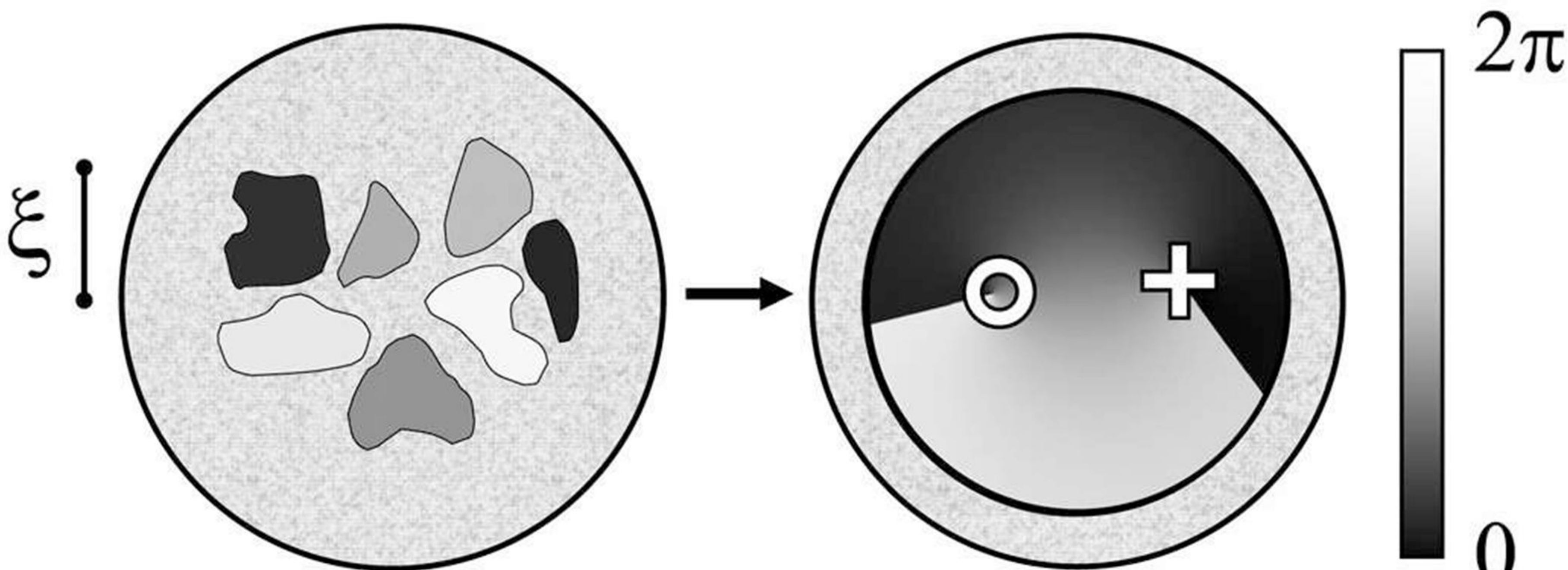
Vortices in the sky

Cosmic strings



Krusius, 2006

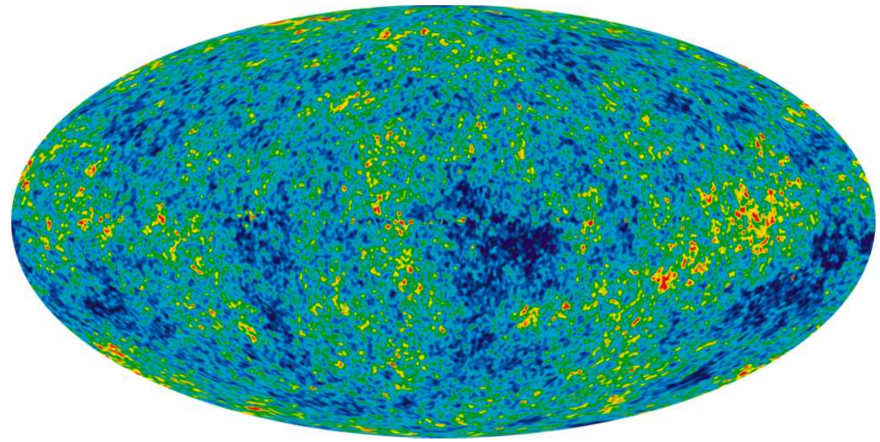
Generation of Structure



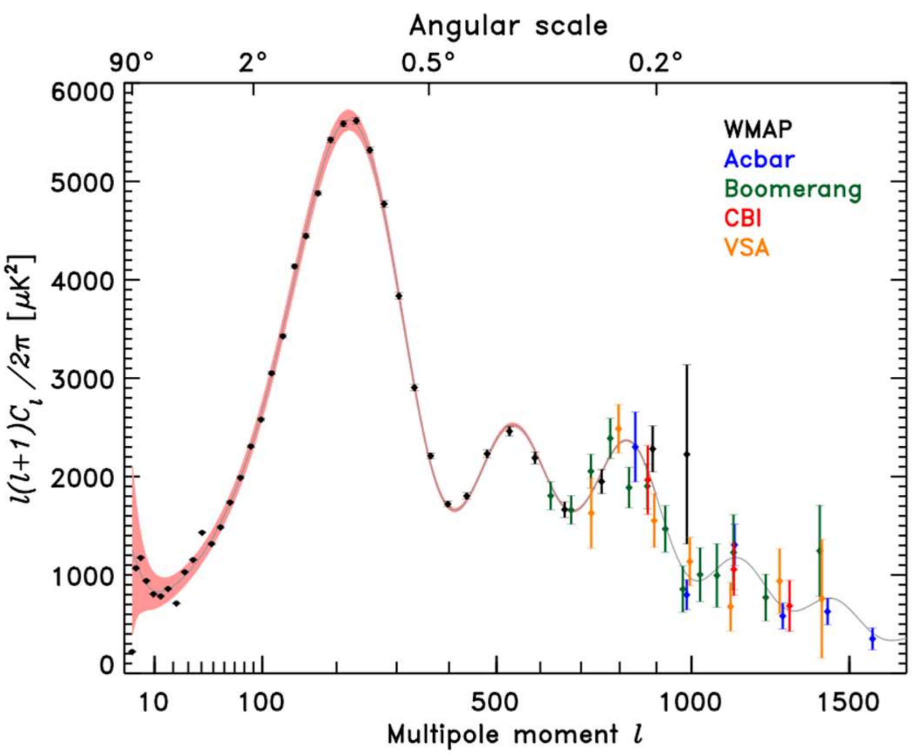
Weyler, Nature 2008

No evidence so far !

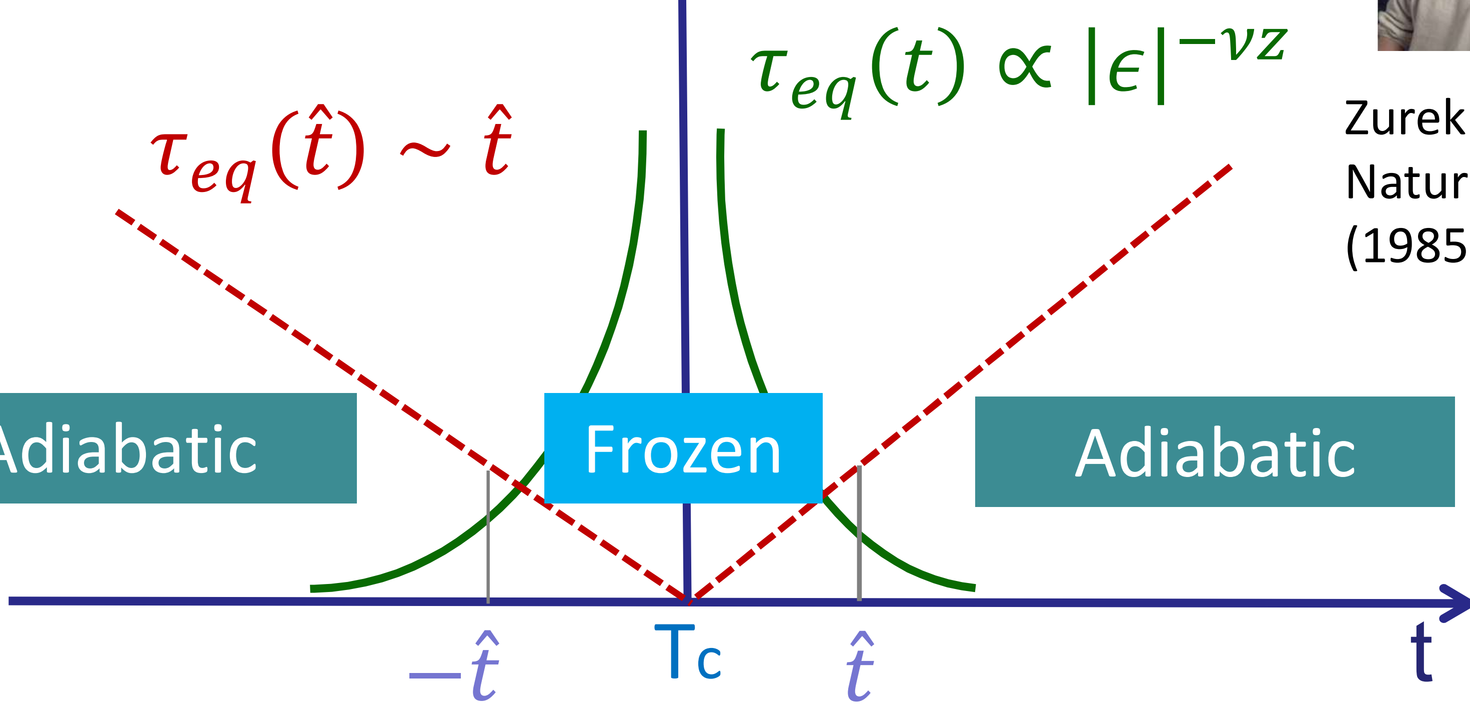
CMB, galaxy distributions…



NASA/WMAP



$$1-\frac{T(t)}{T_c}=\epsilon(t)=t/\tau_Q$$



Zurek
Nature 317
(1985) 505

$$\hat{\xi}=\xi_0|\hat{\epsilon}|^{-\nu}=\xi_0\left(\tau_Q/\tau_0\right)^{\nu/(1+\nu z)}$$

Kibble-Zurek mechanism

$$\rho \sim \hat{\xi}^{-d} \sim \tau_Q{}^{-d\nu/(1+\nu z)}$$

nature COMMUNICATIONS

ARTICLE

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Observation of the Kibble–Zurek scaling law for defect formation in ion crystals

S. Ulm[1], J. Roßnagel[1], G. Jacob[1], C. Degünther[1], S.T. Dawkins[1], U.G. Poschinger[1], R. Nigmatullin[2,3], A. Retzker[4], M.B. Plenio[2,3], F. Schmidt-Kaler[1] & K. Singer[1]

nature Vol 455|16 October 2008|doi:10.1038/nature07334

LETTERS

Spontaneous vortices in the formation of Bose–Einstein condensates

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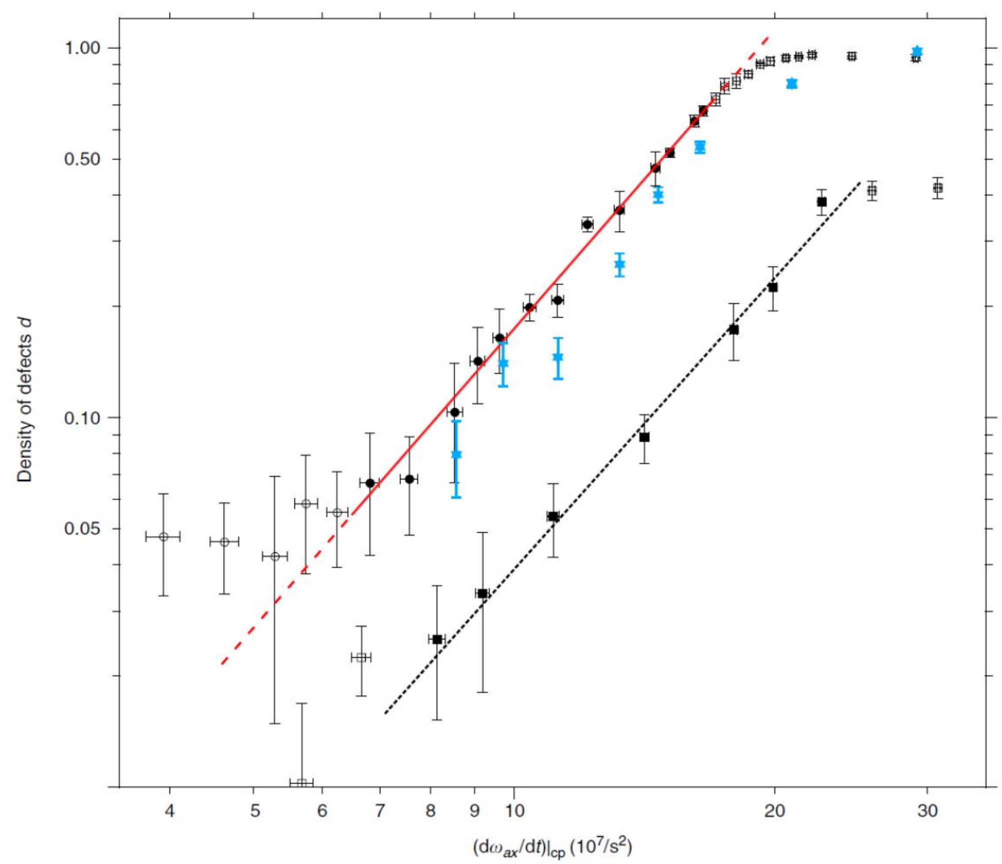
KZ scaling with the quench speed

Too few defects



Issues with KZ | Too many defects

Adiabatic at t_{freeze}?

Defects without a condensate?

$t_{eq} > t > t_{freeze}$ is relevant

Phys. Rev. X 5, 021015 (2015)

Chesler, AGG, Liu

Slow Quenches	Linear response
$\mathbf{t > t_{freeze}}$	Scaling

KZ	Frozen	Adiabatic	
US	Frozen	Coarsening	Adiabatic

t_{freeze} $\quad$ t_{eq}

$$\frac{t_{eq}}{t_{freeze}} \sim (\log R)^{\frac{1}{1+\nu z}}$$

$$|\psi|^2(t) \propto e^{a_2 \bar{t}^{1+z\nu}}$$

$$|\psi|^2(\epsilon) \propto \epsilon^{2\beta}$$

$$\Lambda = (d-z)\nu - 2\beta$$

$$R \sim \xi^{-1} {\tau_Q}^{\Lambda/1+\nu z}$$

$$\gamma = \frac{1+(z-2)\nu}{2(1+z\nu)}$$

$$\rho(t_{eq}) \sim [\log R]^{\gamma} \rho_{KZ}$$

Non adiabatic growth after t_{freeze}

$$C(t,\boldsymbol{r}) \equiv \langle \psi^*(t,\boldsymbol{x}+\boldsymbol{r})\psi(t,\boldsymbol{x})\rangle$$

$$\psi(t,\boldsymbol{q}) = \int dt' G_{\mathrm{R}}(t,t',q)\varphi(t,\boldsymbol{q})$$

$$\langle \varphi^*(t,\boldsymbol{x})\varphi(t',\boldsymbol{x}')\rangle = \zeta\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}')$$

$$G_R(t,t',q) = \theta(t-t')H(q)e^{-i\int_{t'}^{t} dt''\mathfrak{w}_0(\epsilon(t''),q)}$$

$$C(t,q) = \int dt'\, \zeta |G_R(t,t',q)|^2$$

Linear response $t > t_{freeze}$

$|\partial_t \log \mathfrak{w}_0| < |\mathfrak{w}_0|$

$$C(t,q) = \int_{t_{\rm freeze}}^{t} dt' \zeta |H(q)|^2 e^{2\int_{t'}^{t} dt'' {\rm Im}\, \mathfrak{w}_0(\epsilon(t''),q)} + \cdots$$

$$\mathfrak{w}_0(\epsilon, q) = \epsilon^{z\nu} h(q\epsilon^{-\nu})$$

$${\rm Im}\,\mathfrak{w}_0 = -a\epsilon^{(z-2)\nu} q^2 + b\epsilon^{z\nu} + \ldots,$$

$q_{max} \sim \epsilon(t)^{\nu}$

${\rm Im}\,\mathfrak{w}_0 > 0$

Unstable Modes → Growth

$\langle \psi(t) \rangle \;\; t > t_{freeze}$

Protocol

$$\epsilon(t) = t/\tau_Q \qquad t_i = (1 - T_i/T_c)\tau_Q < 0$$

$$t \in (t_i, t_f) \qquad t_f = (1 - T_f/T_c)\tau_Q > 0$$

Slow quenches

$t_f \geq t_{eq}$

$t > t_{freeze}$

Correlation length increases

$$C(t,r) \sim |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}} \qquad \bar{t} \equiv \frac{t}{t_{\rm freeze}}$$

$$\ell_{\rm co}(\bar{t}) = a_3 \xi_{\rm freeze} \bar{t}^{\frac{1+(z-2)\nu}{2}}$$

Condensate growth

Stoof, J. Low Temp. Phys. 114, 11 (1999), 124, 431 (2001)

$$|\psi|^2(t) \sim \tilde{\varepsilon}(t) e^{a_2 \bar{t}^{1+z\nu}}$$

$$\tilde{\varepsilon}(t) \equiv \zeta t_{\rm freeze} \ell_{\rm co}^{-d}(t)$$

Adiabatic evolution $t = t_{eq} \gg t_{freeze}$

$$|\psi|^2(t = t_{\rm eq}) \sim |\psi|^2_{\rm eq}(\epsilon(t_{\rm eq}))$$

Defects

$$\rho(t_{\rm eq}) \sim 1/\ell_{\rm co}^{d-D}(t_{\rm eq}) \sim \left[\log(\zeta^{-1}\tau_Q^{\Lambda})\right]^{-\frac{(d-D)(1+(z-2)\nu)}{2(1+z\nu)}} \rho_{\rm KZ}$$

Fast quenches

$$t_f \ll t_{eq}$$

$$q_{max}(T_f) = \epsilon(t_f)^{d\nu}$$

Breaking of τ_Q scaling

KZ $\quad t_f < t_{freeze}$

US $\quad t_{freeze} \ll t_f \ll t_{eq}$

Exponential growth

$$|\psi|^2(t) \sim \epsilon_f^{(d-z)\nu} \zeta \exp\left[2b(t - t_{\mathrm{freeze}})\epsilon_f^{\nu z}\right]$$

Number of defects

Independent of τ_Q

$$\rho \sim \begin{cases} \epsilon_f^{(d-D)\nu} & R_f \lesssim O(1) \\ \epsilon_f^{(d-D)\nu} \log^{-\frac{d-D}{2}} R_f & R_f \gg 1 \end{cases}$$

$$R_f \equiv \frac{\epsilon_f^{2\beta}}{\zeta \epsilon_f^{(d-z)\nu}} \qquad \epsilon_f \equiv \frac{T_c - T_f}{T_c}$$

Holography?

Defects survive large N limit

Universality

Real time

Dual gravity theory

$$S_{\text{grav}} = \frac{1}{16\pi G_{\text{Newton}}} \int d^4x \sqrt{-G} \left[R + \Lambda + \frac{1}{e^2} \left(-\frac{1}{4} F_{MN} F^{MN} - |D\Phi|^2 - m^2 |\Phi|^2 \right) \right]$$

$$\Lambda = -3 \qquad m^2 = -2$$

Herzog, Horowitz, Hartnoll, Gubser

AdS_4

$$ds^2 = r^2 g_{\mu\nu}(t, \boldsymbol{x}, r) dx^\mu dx^\nu + 2 dr dt$$

Eddington-Finkelstein coordinates

Probe limit

$$0 = \nabla_M F^{NM} - J^M,$$

$$0 = (-D^2 + m^2)\Phi.$$

EOM's:

PDE's in x,y,r,t

Boundary conditions:

$r \to \infty$

= 0

$$\Psi = \frac{\psi^{(1)}(x,y,t)}{r} + \frac{\psi^{(2)}(x,y,t)}{r^2} + ..$$

$$A_t = \mu - \rho/r$$

hep-th/9905104v2
1309.1439
Science 2013

Drive:

$$\epsilon(t) = t/\tau_Q \qquad t_i = (1 - T_i/T_c)\tau_Q$$

$$t \in (t_i, t_f) \qquad t_f = \left(1 - T_f/T_c\right)\tau_Q$$

No solution of Einstein equations but do not worry, Hubeny 2008

Dictionary:

$$\langle O_2 \rangle \sim \psi_2$$

Stochastic driving

$$\psi^{(1)} = \varphi(t, x)$$

$$\langle \varphi^*(t, x)\varphi(t', x')\rangle = \xi\delta(t - t')\delta(x - x')$$

Field theory:

$$\xi(T, \nu)$$

Quantum/thermal fluctuations

Gravity:

$$\xi \propto 1/N^2$$

Hawking radiation

Predictions

Mean field critical exponents

Slow quenches:

$$C(t,r) \sim |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}, \quad |\psi|^2(t) \sim \tilde{\varepsilon} t_{\rm freeze} \bar{t} e^{a_2 \bar{t}^2}, \quad \ell_{\rm co}(t) \sim \xi_{\rm freeze} \sqrt{\bar{t}}$$

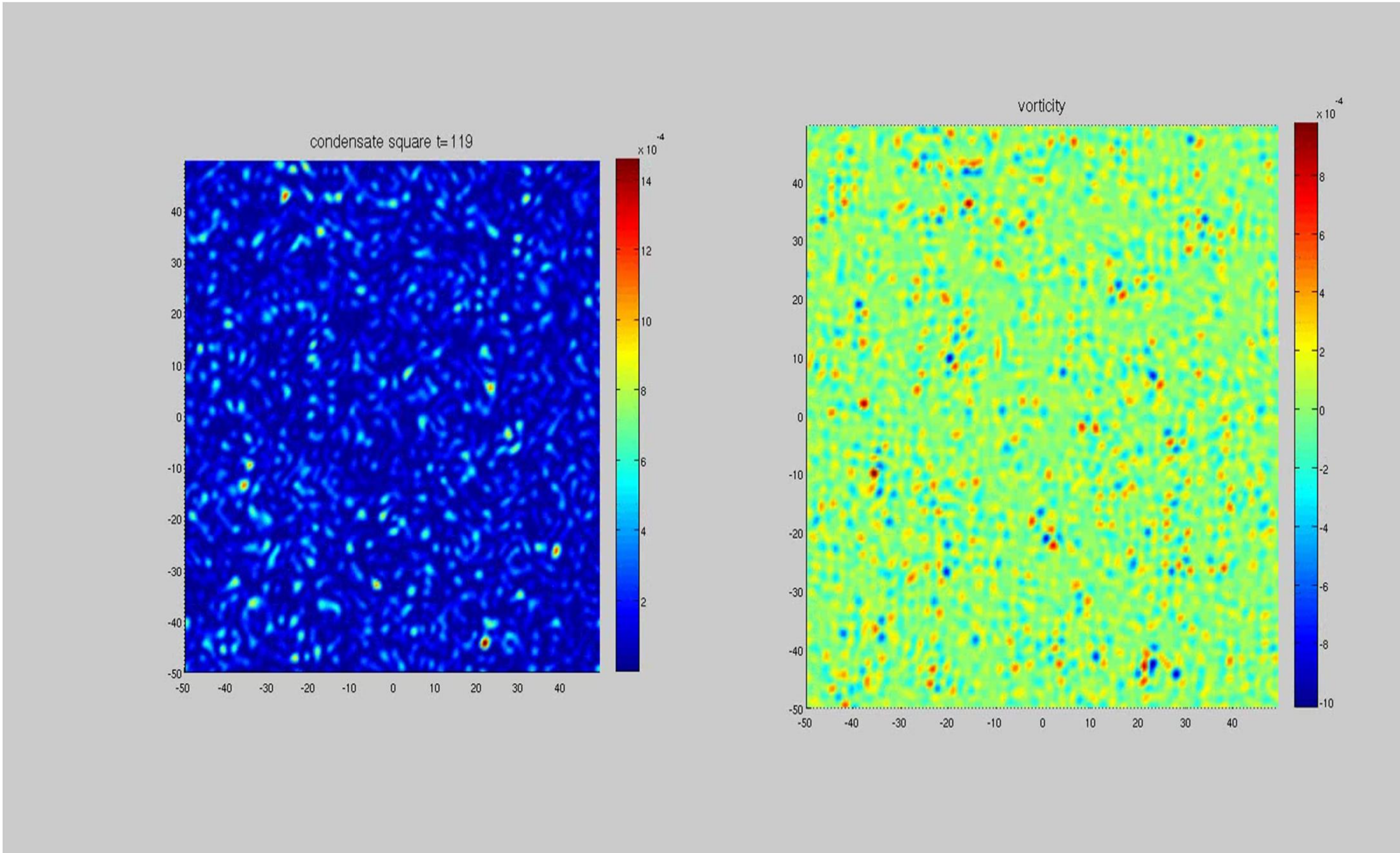
$$\frac{t_{\rm eq}}{t_{\rm freeze}} \sim \sqrt{\log \frac{N^2}{\sqrt{\tau_Q}}} \qquad \rho \sim \frac{1}{\sqrt{\log \frac{N^2}{\sqrt{\tau_Q}}}} \rho_{\rm KZ}$$

Fast quenches:

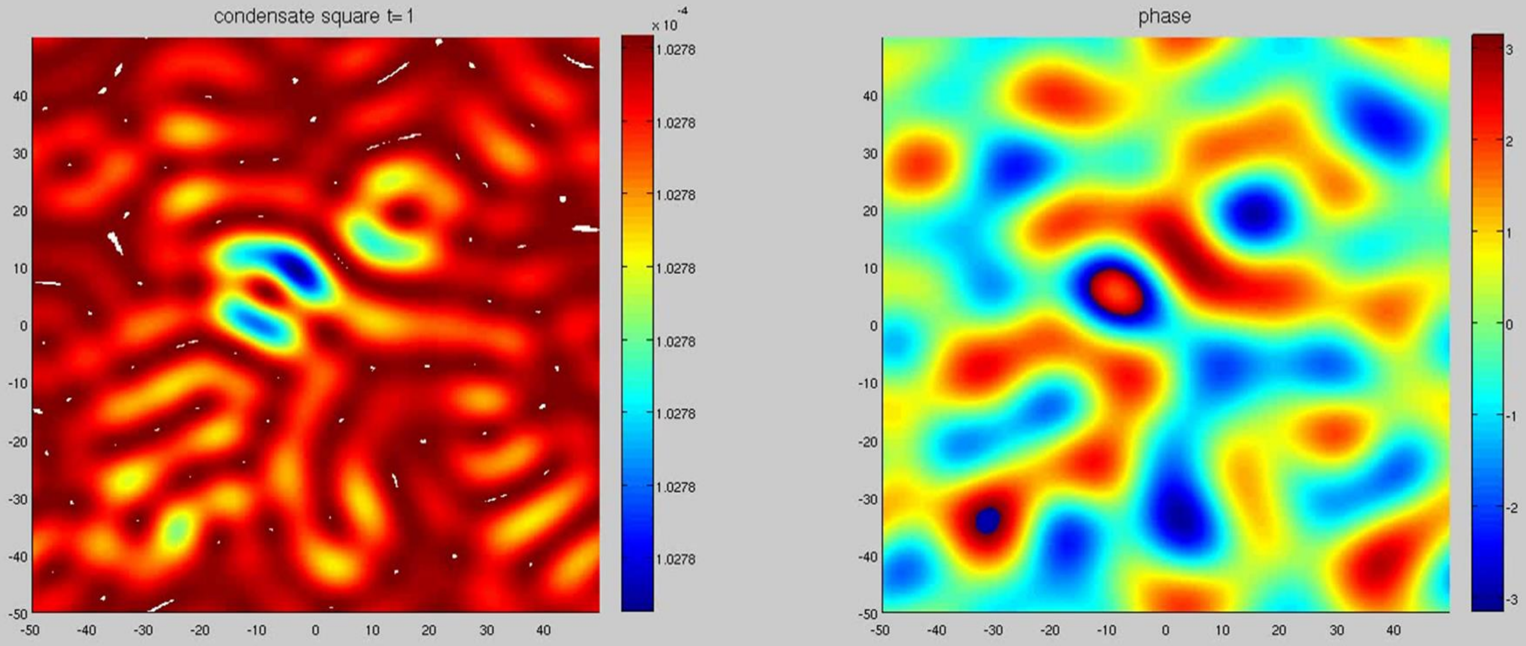
$$C(t,r) = |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}, \qquad |\psi|^2(t) \sim \zeta \exp\left[2b(t - t_{\rm freeze})\epsilon_f\right]$$

$$\ell_{\rm co}^2(t) = 4a(t - t_{\rm freeze}) \qquad \rho \sim \frac{\epsilon_f}{\log \frac{N^2}{\epsilon_f}}$$

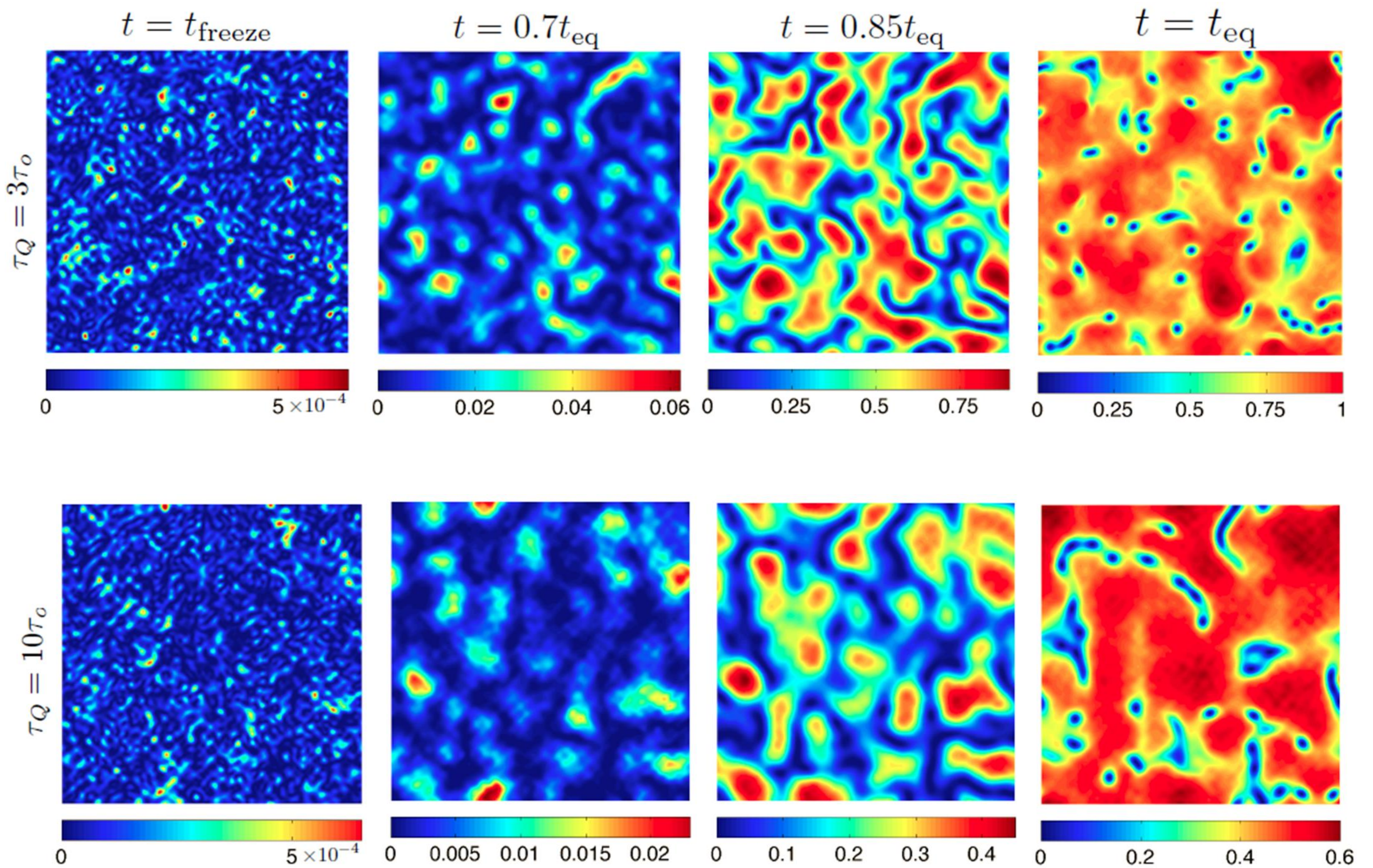
Movies!!



condensate square t=119
x 10^-4
vorticity
x 10^-4



Slow

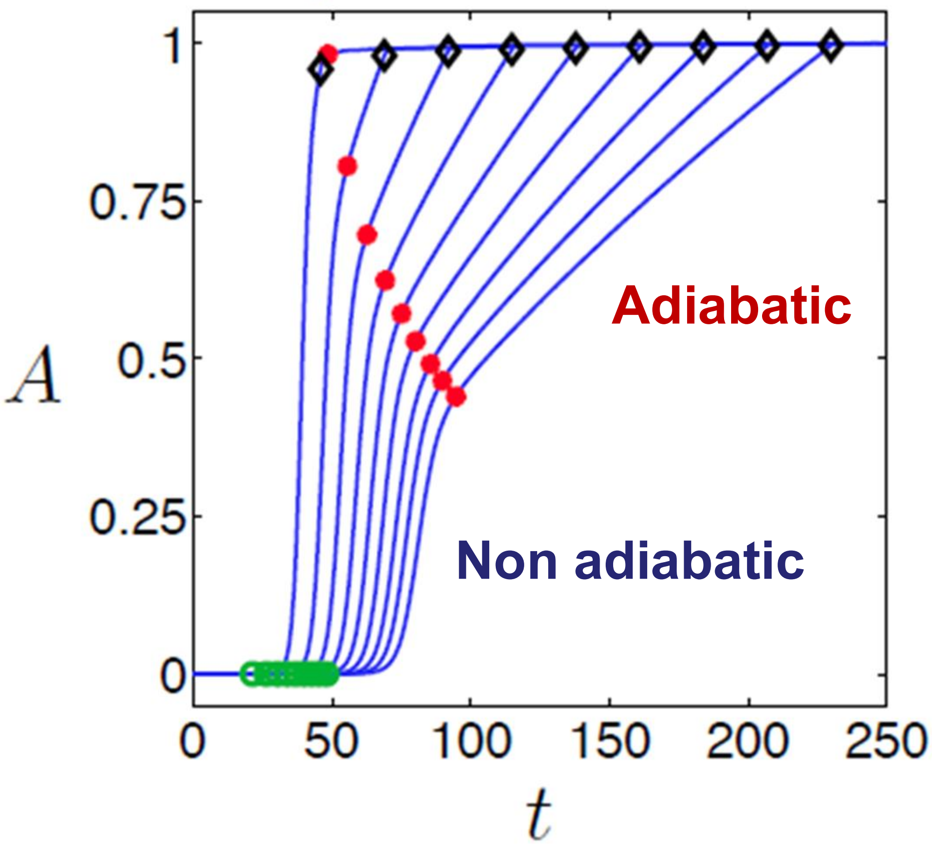


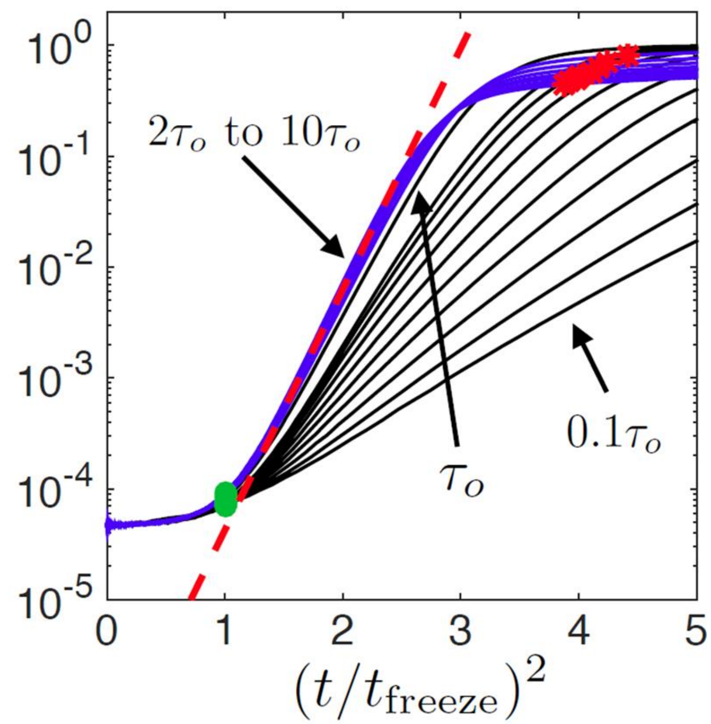
t_{eq} *is the relevant scale*

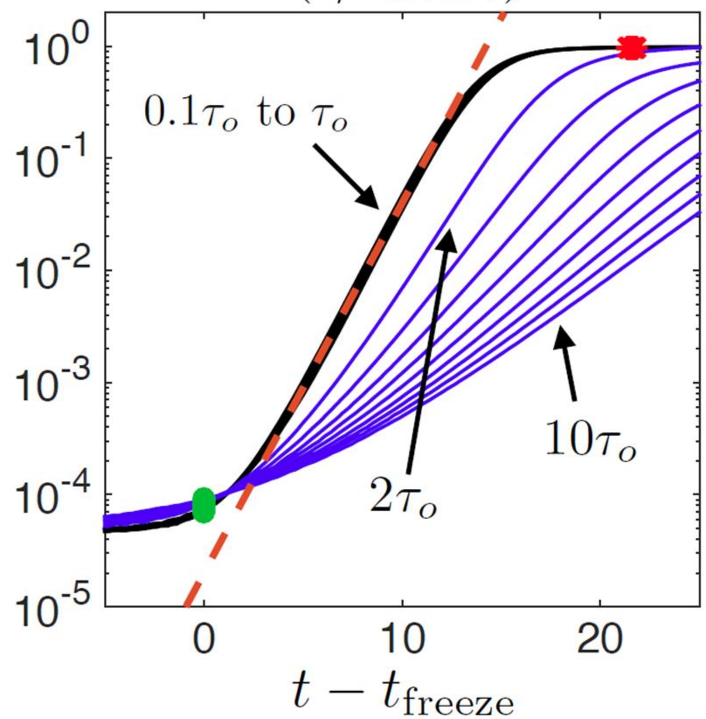
$$A(t) = \frac{1}{M} \sum_{i=1}^{M} \frac{a_i(t)}{a_i(\infty)}$$

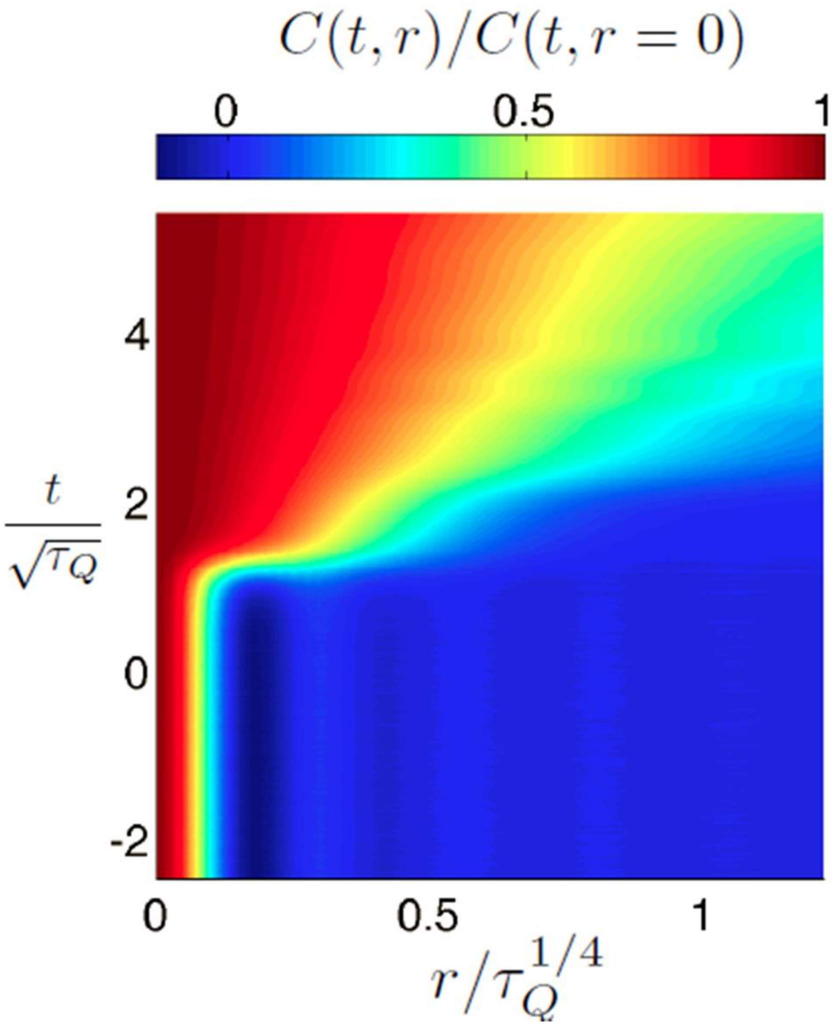
$$a_i(t) \equiv \int d^2x \, |\psi_i(t, \boldsymbol{x})|^2$$

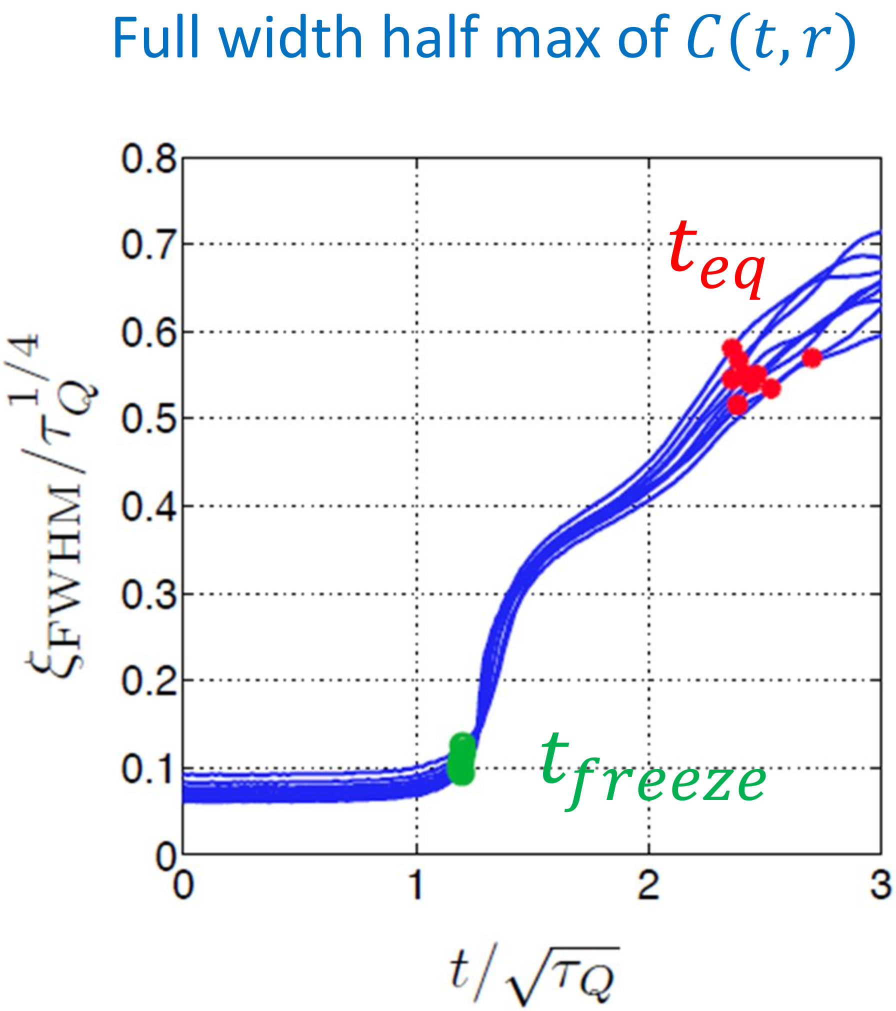
$\tau_Q = 2\tau_o, \ldots, 10\tau_o$











Strong coarsening

$t > t_{freeze}$

$$\ell_{co}(t) \sim \xi_{freeze}\sqrt{t/t_{freeze}}$$

Slow

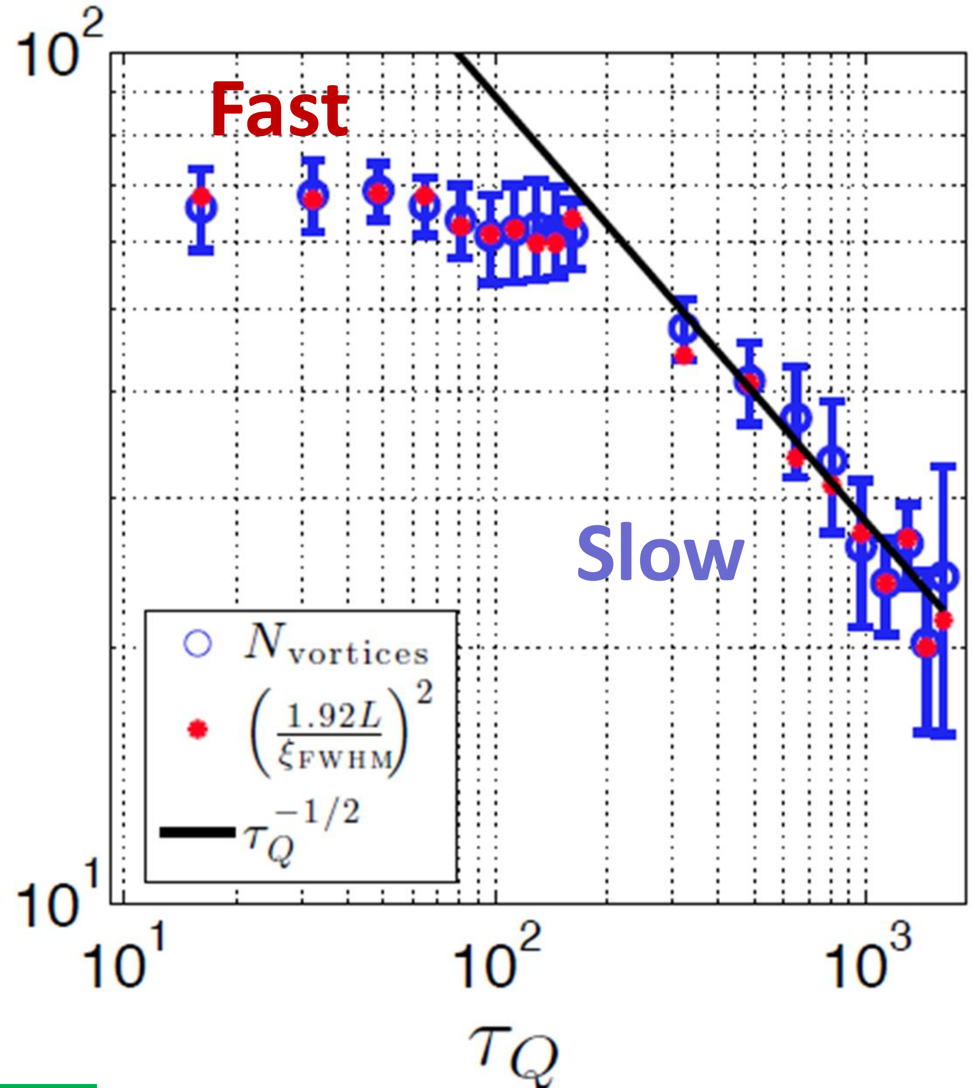
$$\rho \sim \frac{\rho_{KZ}}{\left(log\left(N^2/{\tau_Q}^{1/2}\right)\right)^{1/2}}$$

Fast

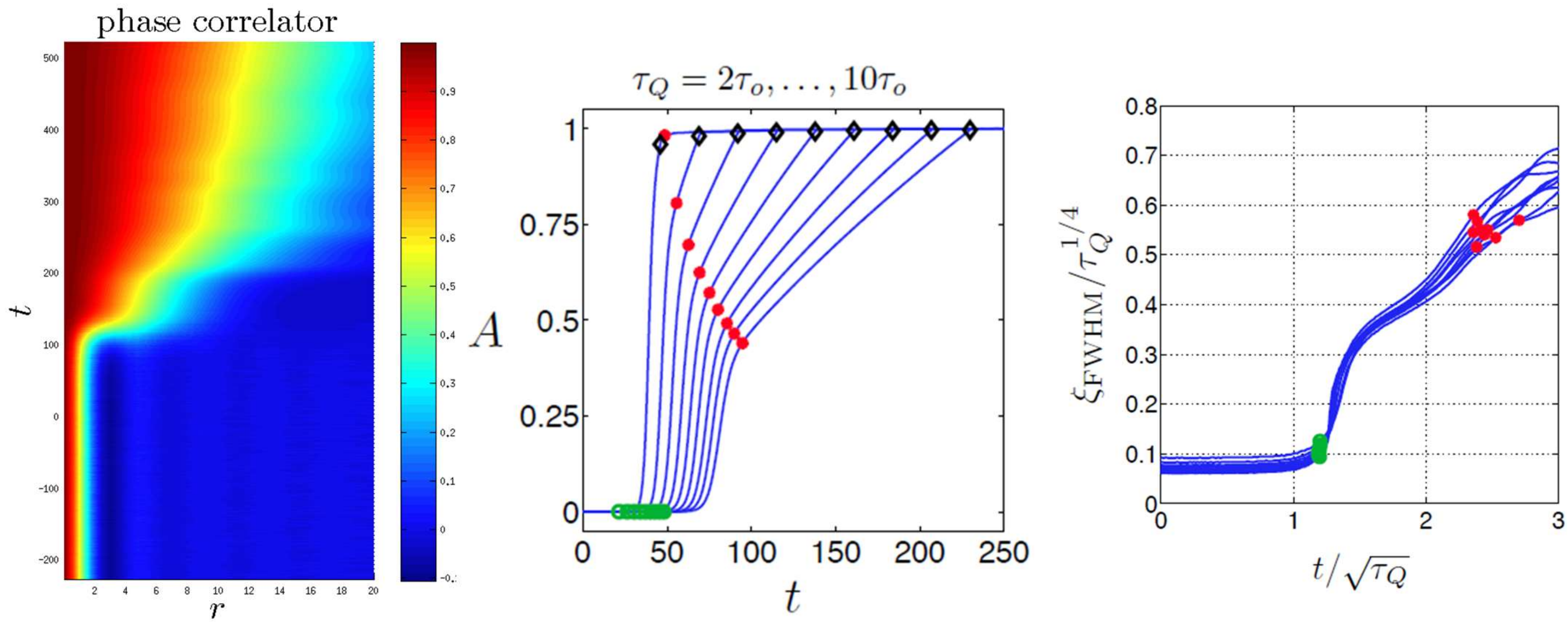
$$\rho \sim \frac{\epsilon_f}{\log(\frac{N^2}{\epsilon_f})}$$

Relevant for ^{4}He ?

$$t_{\rm eq} \sim [\log \tau_Q]^{1/(1+\nu z)} t_{\rm freeze}$$



~25 times less defects than KZ prediction!!



time

Freezing

Condensate formation

Defect generation

Phase coherence ?

Thanks!