UNIVERSITY OF CAMBRIDGE

Ionic conductors

Lecture 7

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Course B: Materials for Devices

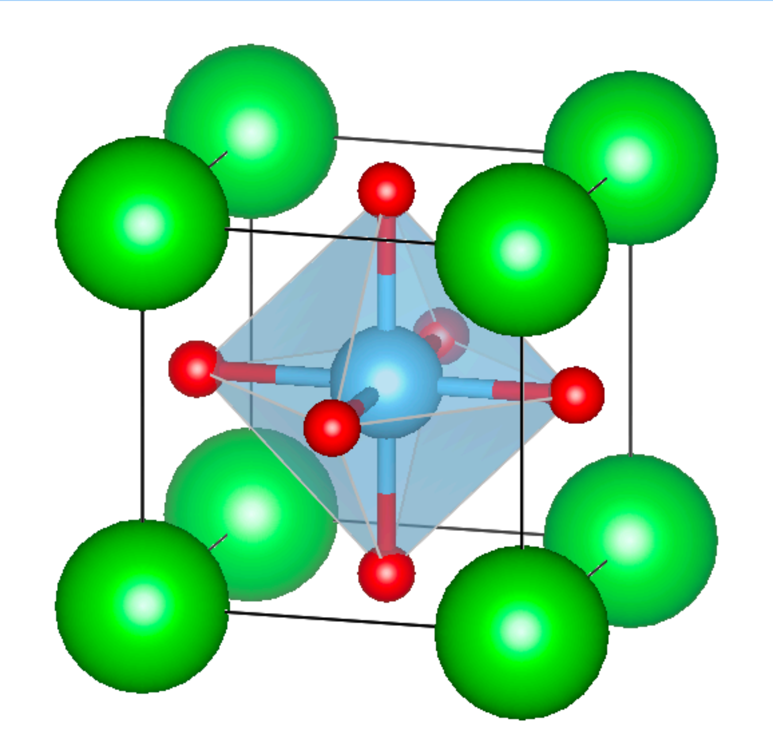
Professor M does Science

http://www.tcm.phy.cam.ac.uk/~bm418/

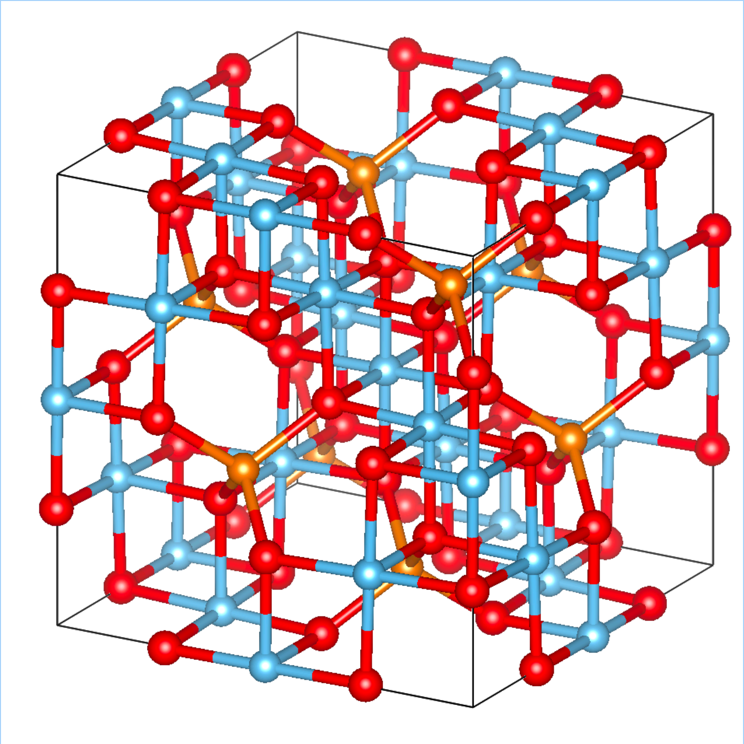
Course B: Materials for Devices

order → disorder

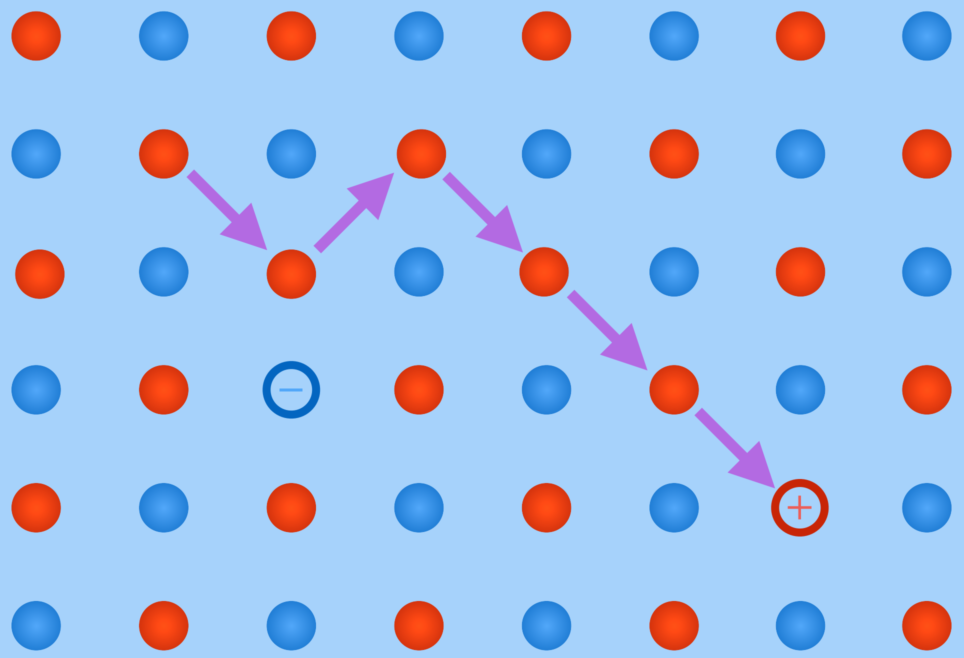
electric polarisation in materials



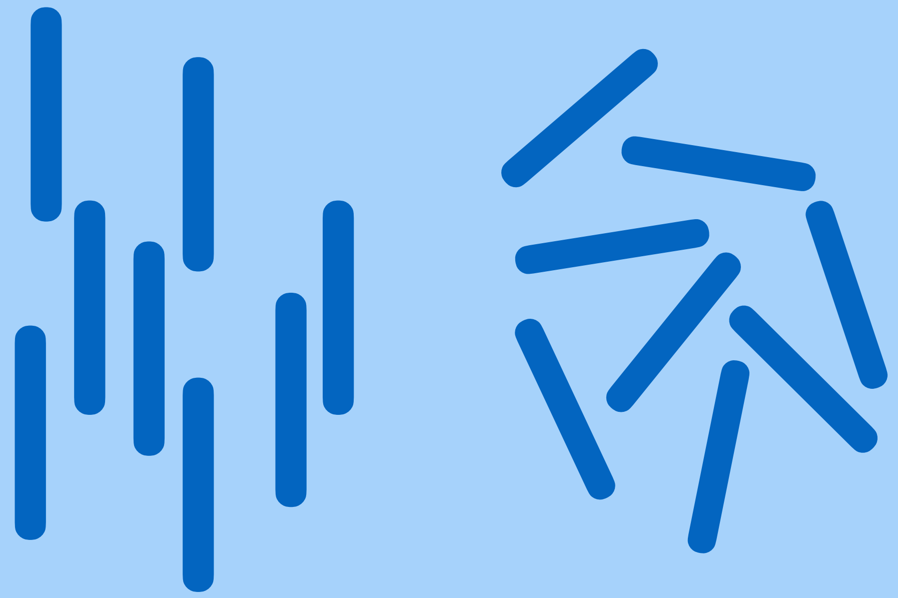
magnetism in materials



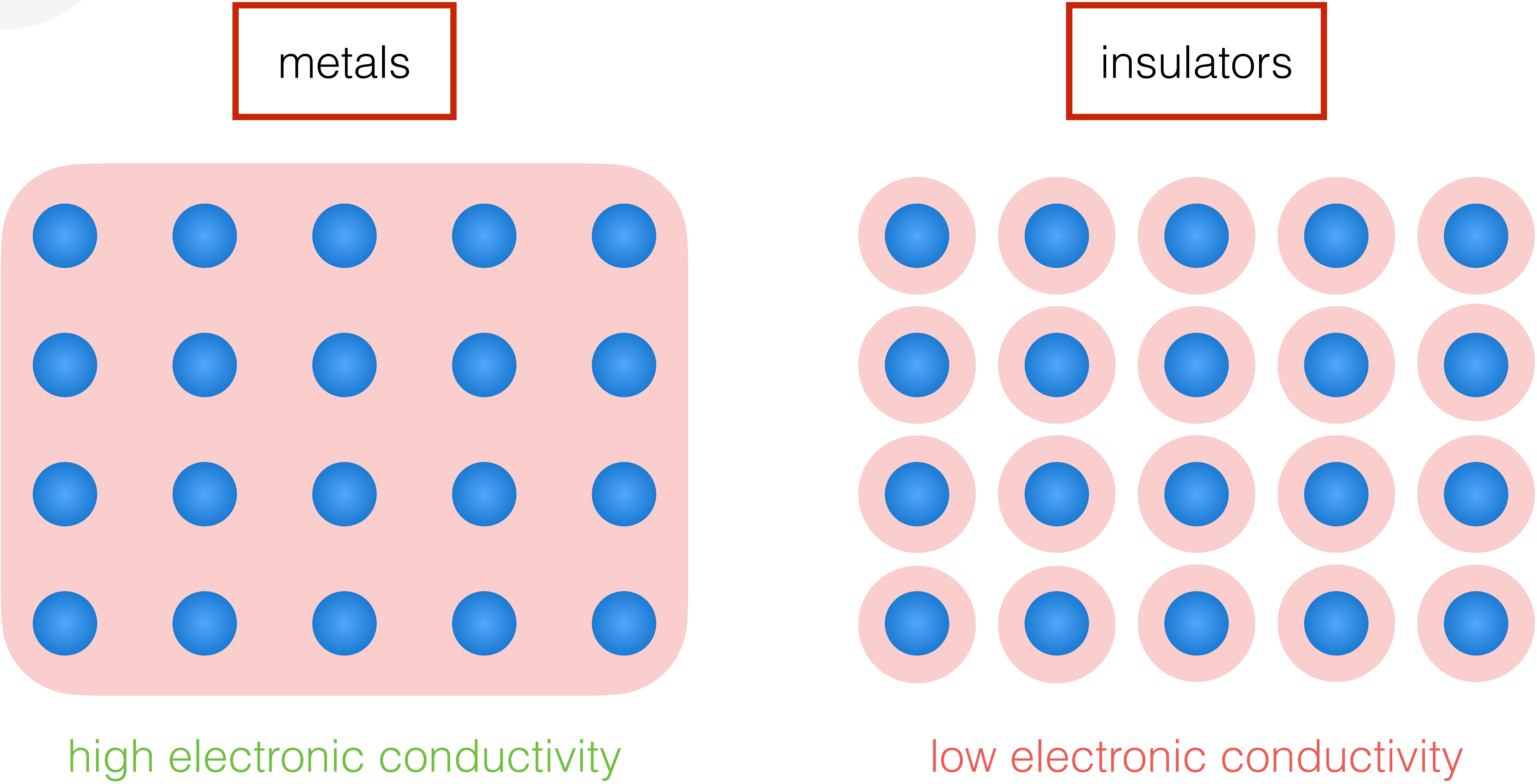
ionic conductors



liquid crystals

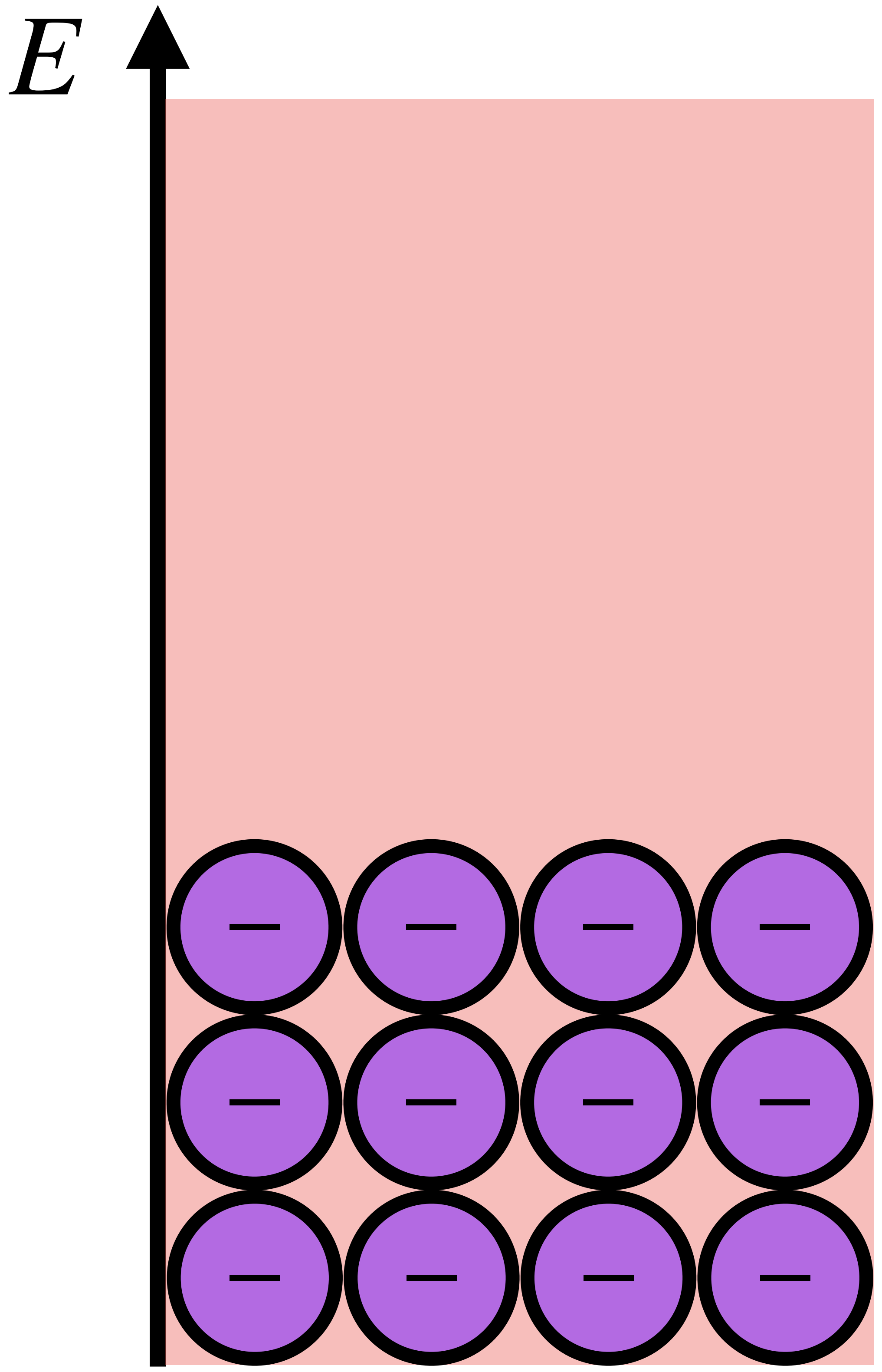


Metals vs insulators



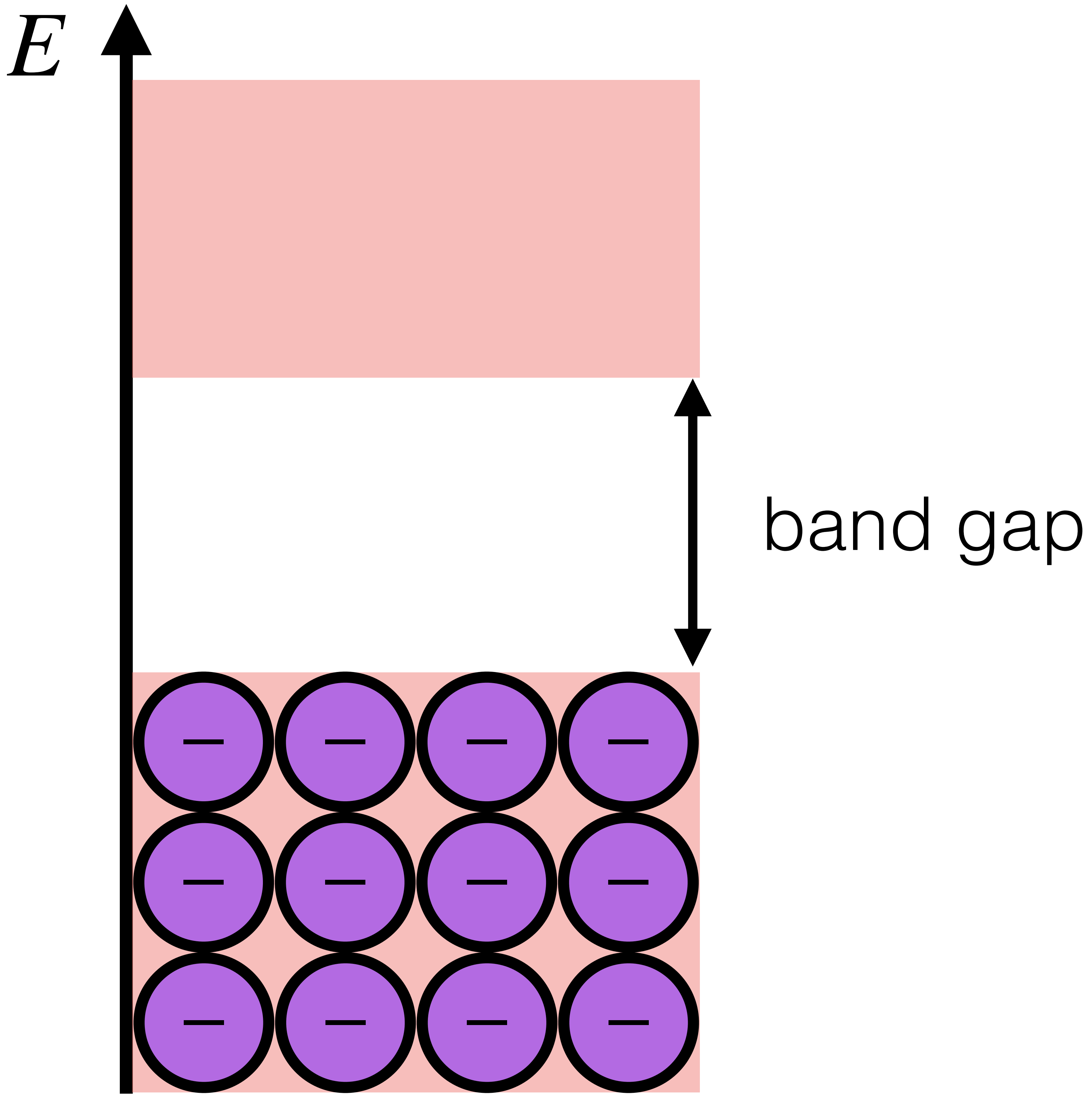
Metals vs insulators

metals



high electronic conductivity

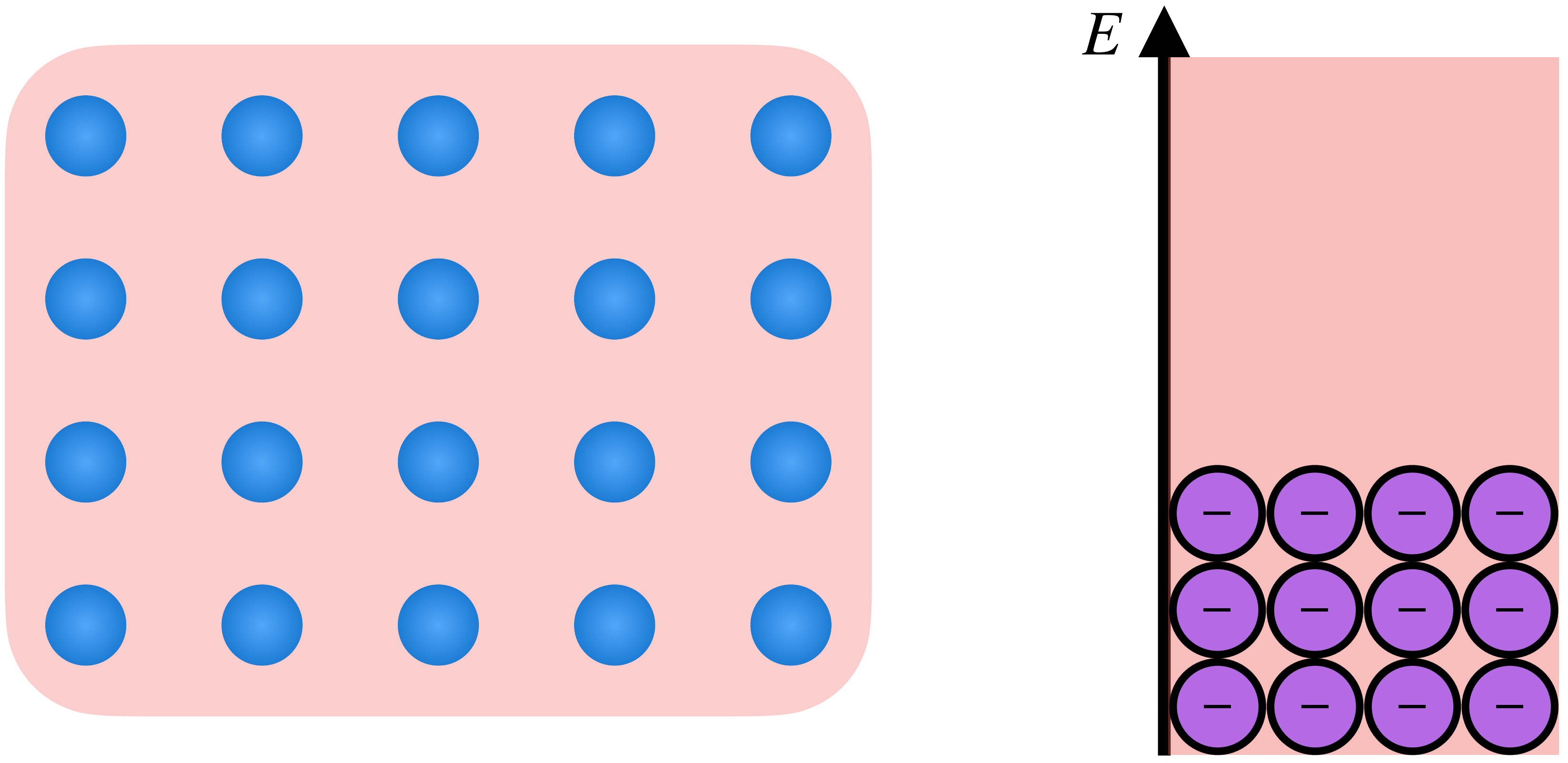
insulators



low electronic conductivity

Metals vs insulators

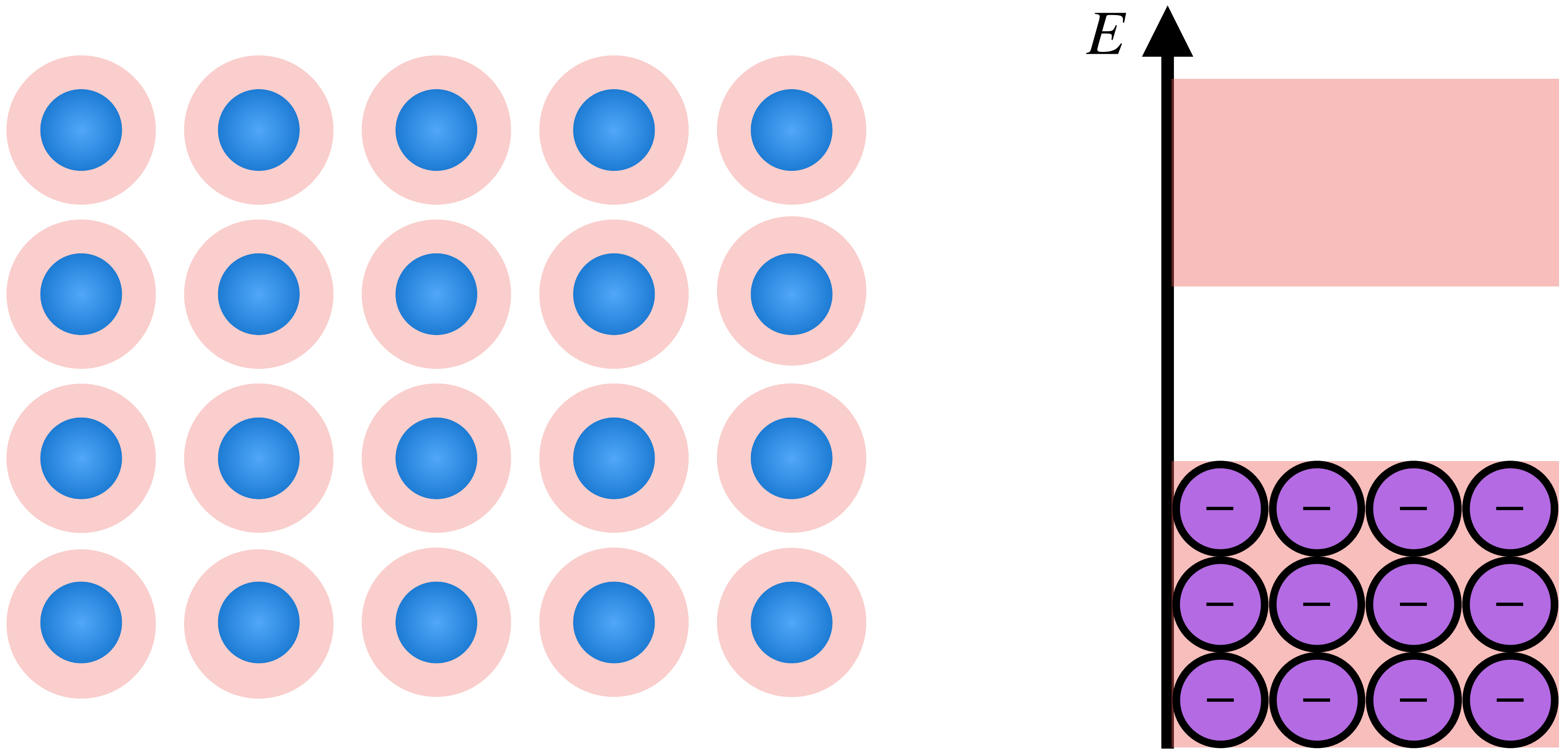
metals



- Silver (Ag): $\sigma = 6.3 \times 10^7 \ \mathrm{Sm}^{-1}$
- Copper (Cu): $\sigma = 6.0 \times 10^7 \ \mathrm{Sm}^{-1}$
- Gold (Au): $\sigma = 4.1 \times 10^7 \ \mathrm{Sm}^{-1}$

high electronic conductivity

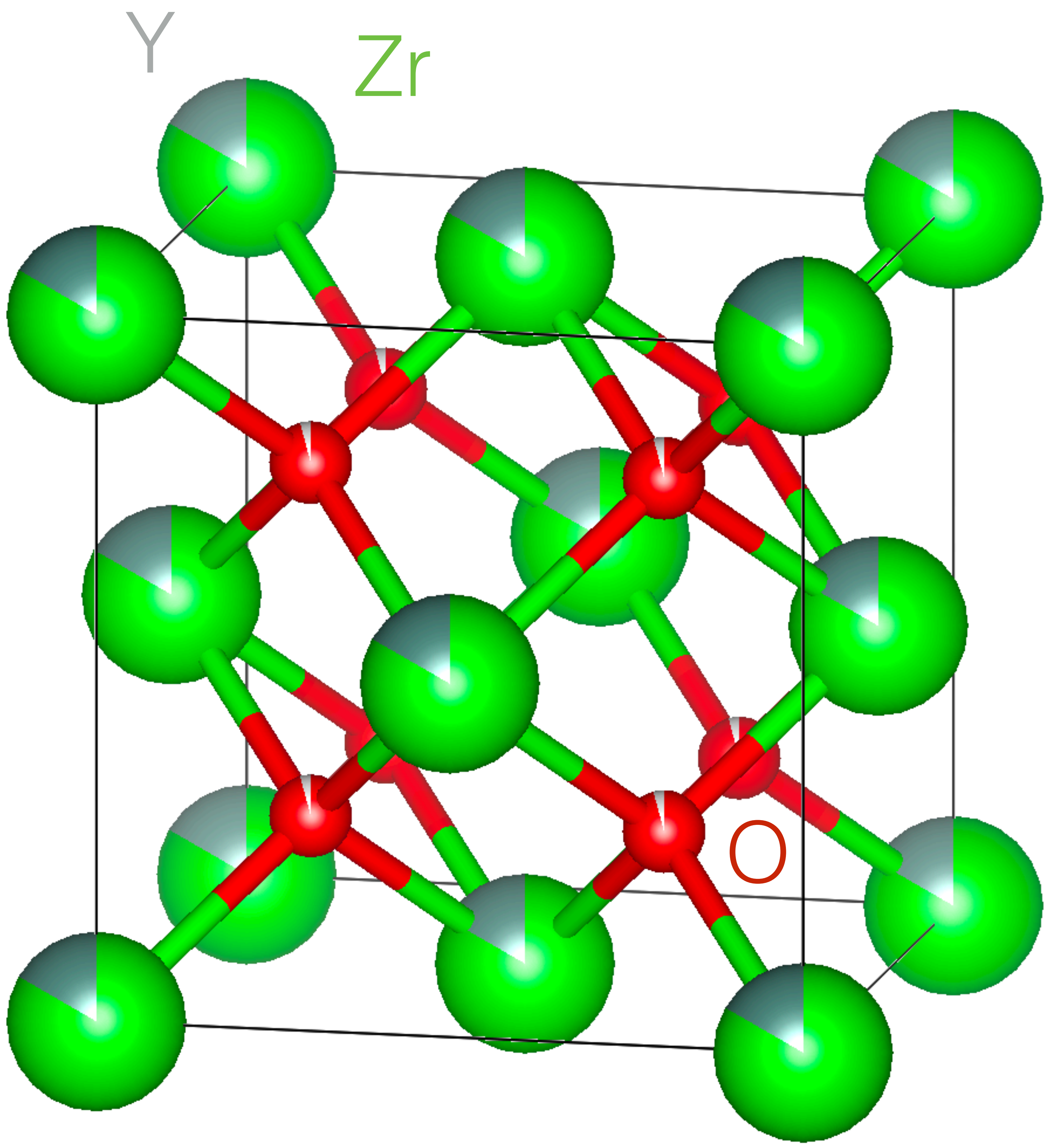
insulators



- Air: $\sigma \simeq 5 \times 10^{-15} \ \mathrm{Sm}^{-1}$
- Glass: $\sigma \simeq 10^{-13} \ \mathrm{Sm}^{-1}$
- Al_2O_3: $\sigma = 10^{-10} \ \mathrm{Sm}^{-1}$

low electronic conductivity

Ionic conduction



- Yttrium stabilised zirconia
- Complex structure (see Lecture 8)
- No conduction electrons (band gap)

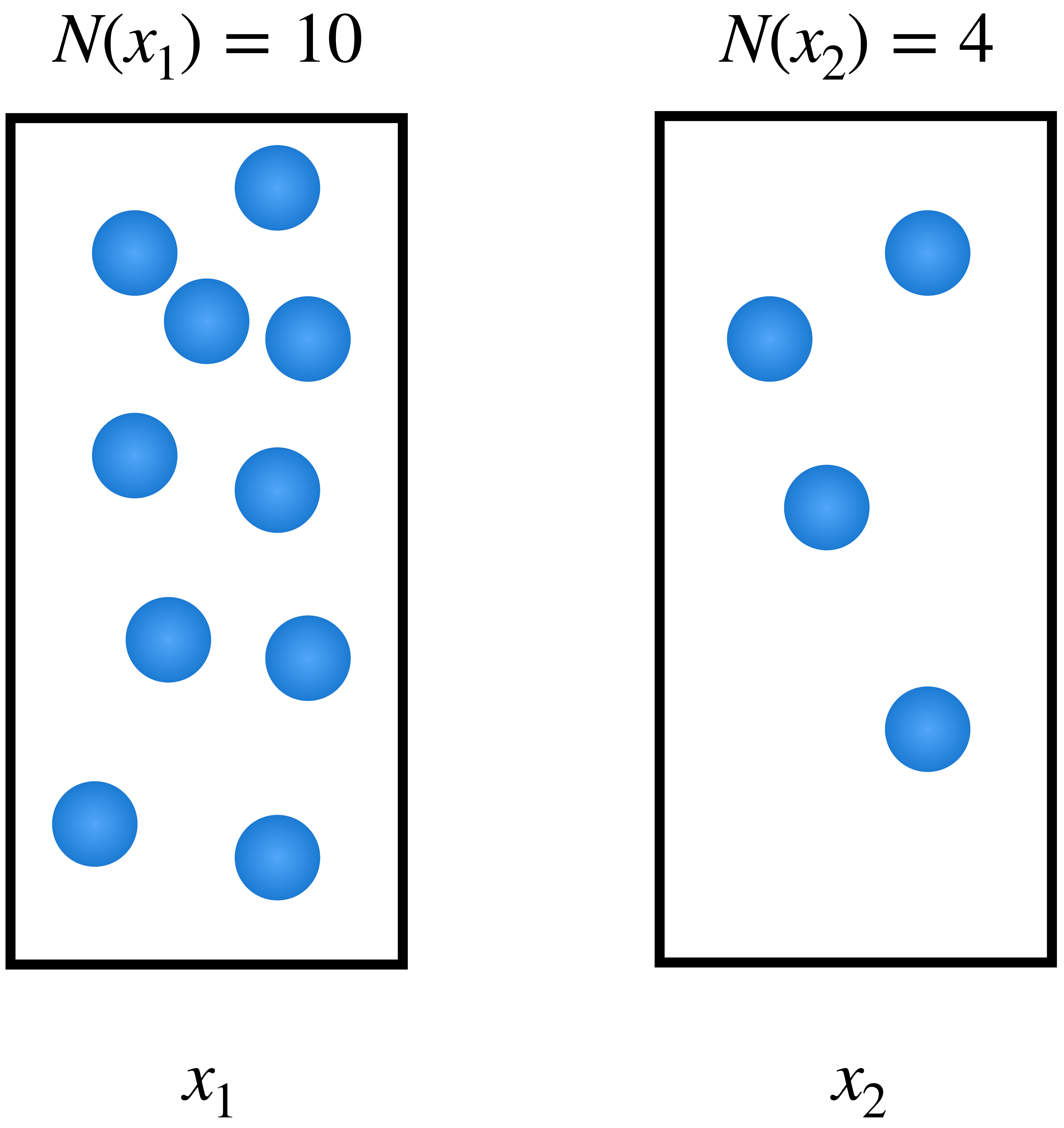
$$\sigma \sim 0.1\ \mathrm{Sm}^{-1}$$

- Copper (Cu): $\sigma = 6.0 \times 10^{7}\ \mathrm{Sm}^{-1}$
- Al_2O_3: $\sigma = 10^{-10}\ \mathrm{Sm}^{-1}$

Random walk and diffusion

- At every time step:

$$P(x_1 \to x_2) = P(x_2 \to x_1) \equiv P$$

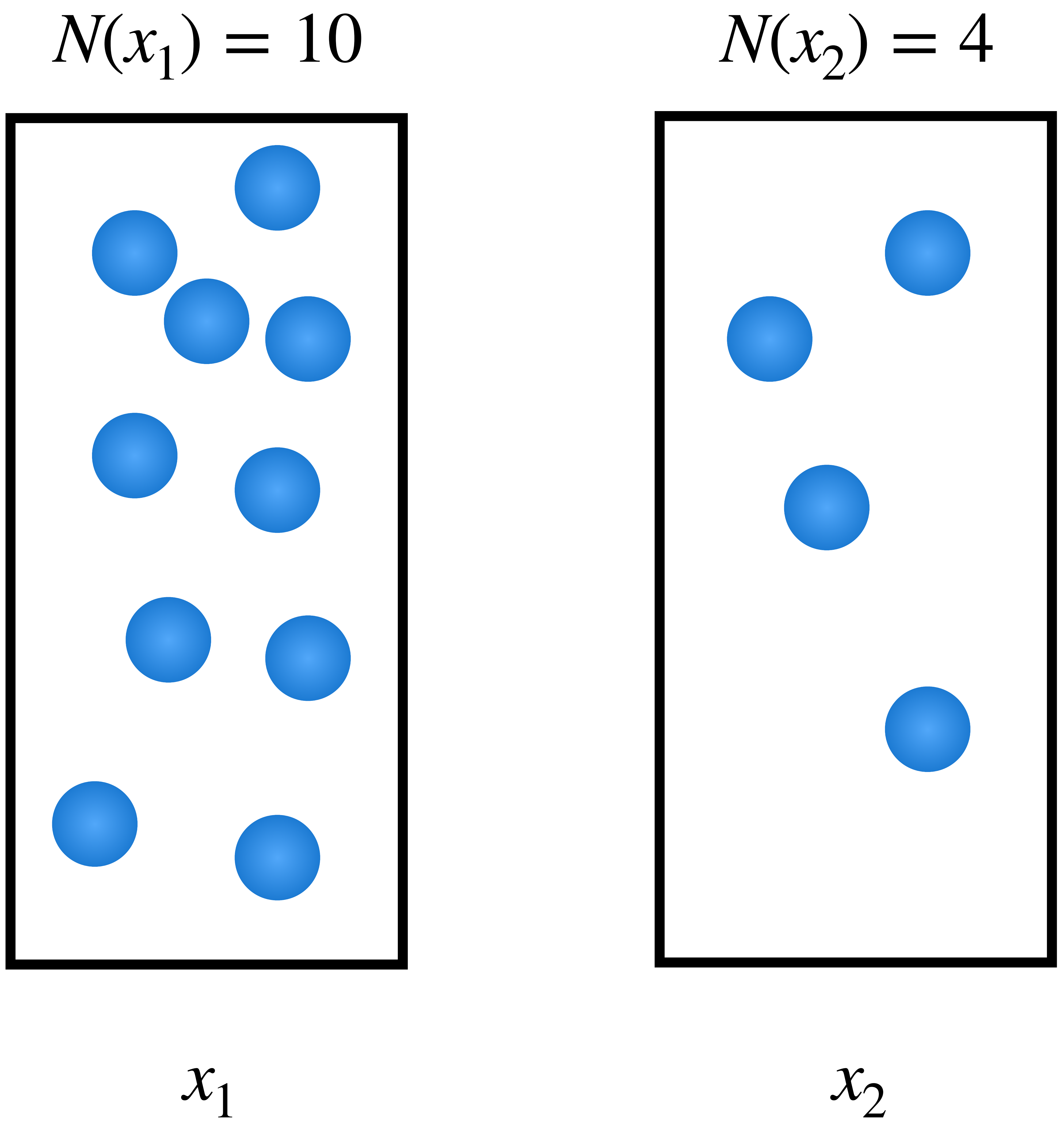


Random walk and diffusion

- At every time step:

$$P(x_1 \to x_2) = P(x_2 \to x_1) \equiv P$$

- Example: $P = 0.5$



Random walk and diffusion

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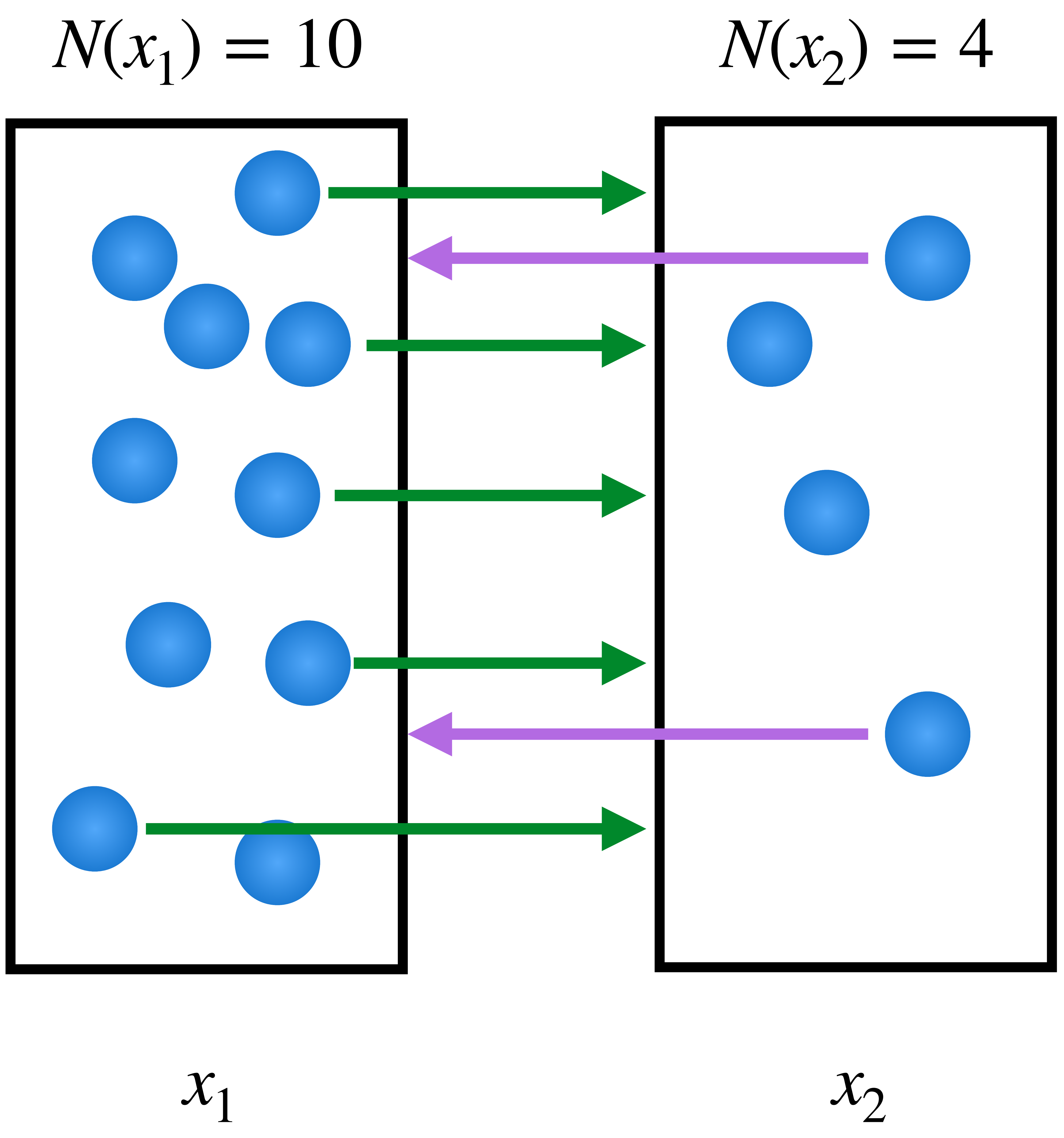

Random walk and diffusion

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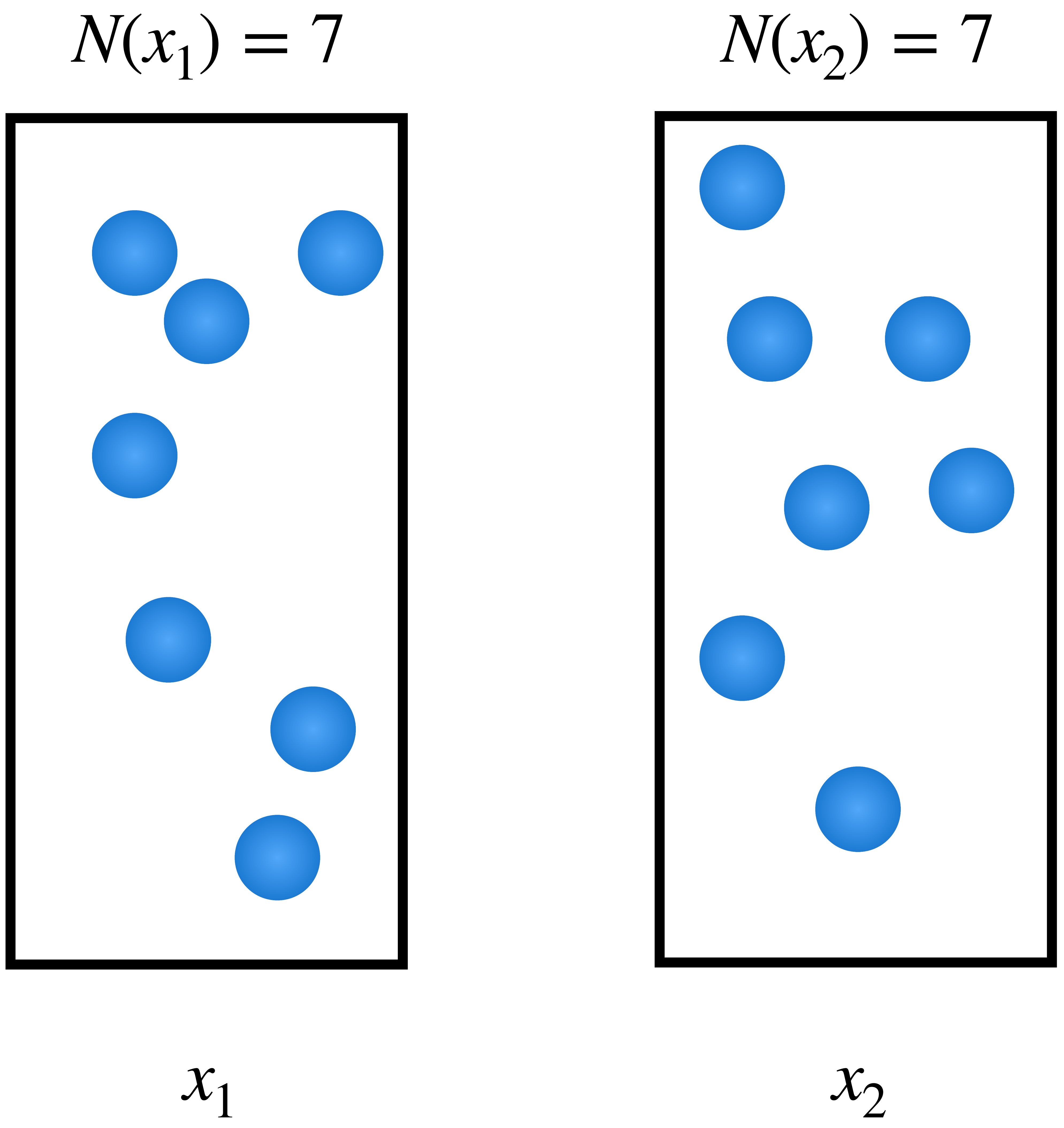


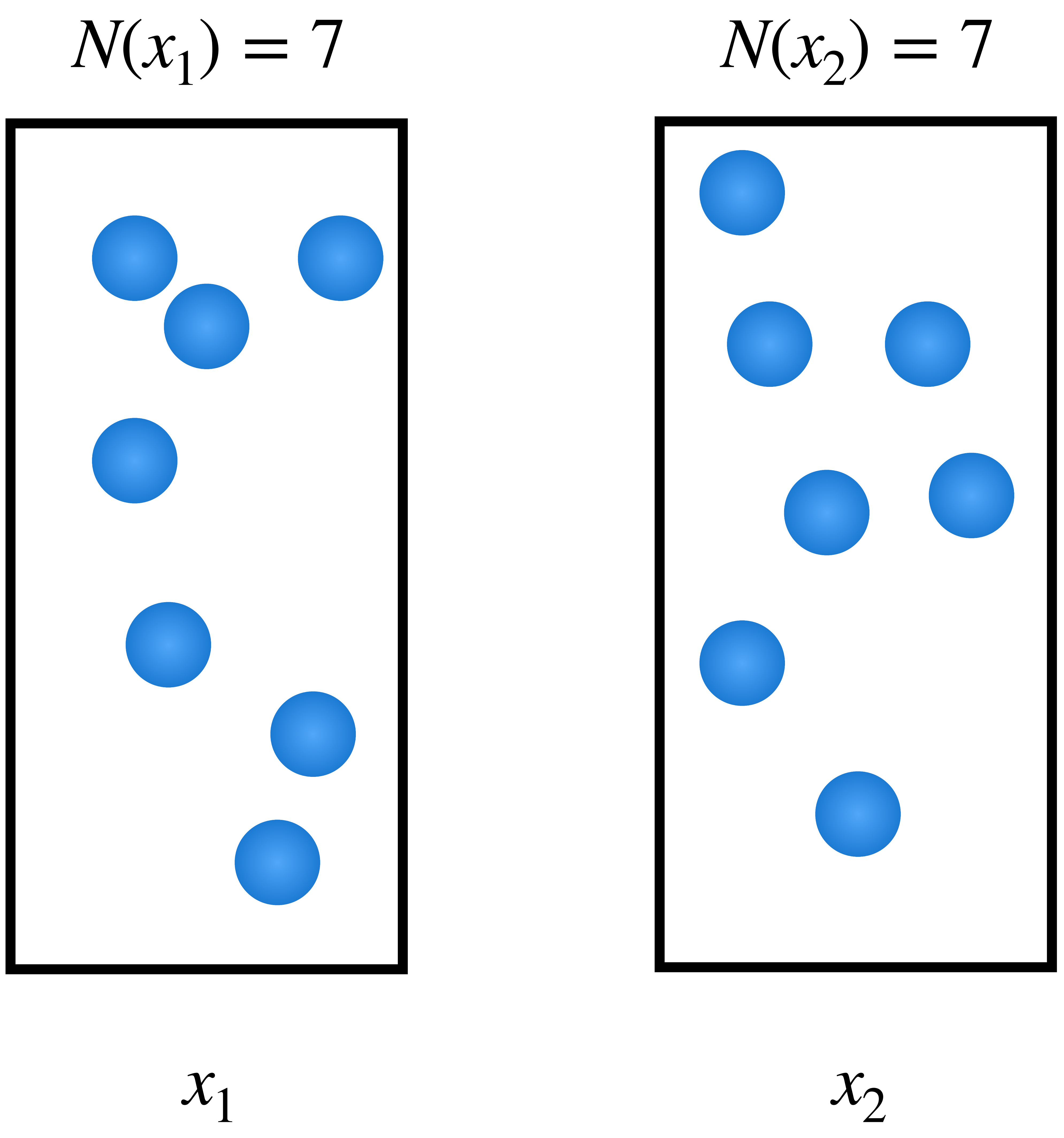
Random walk and diffusion

- At every time step:

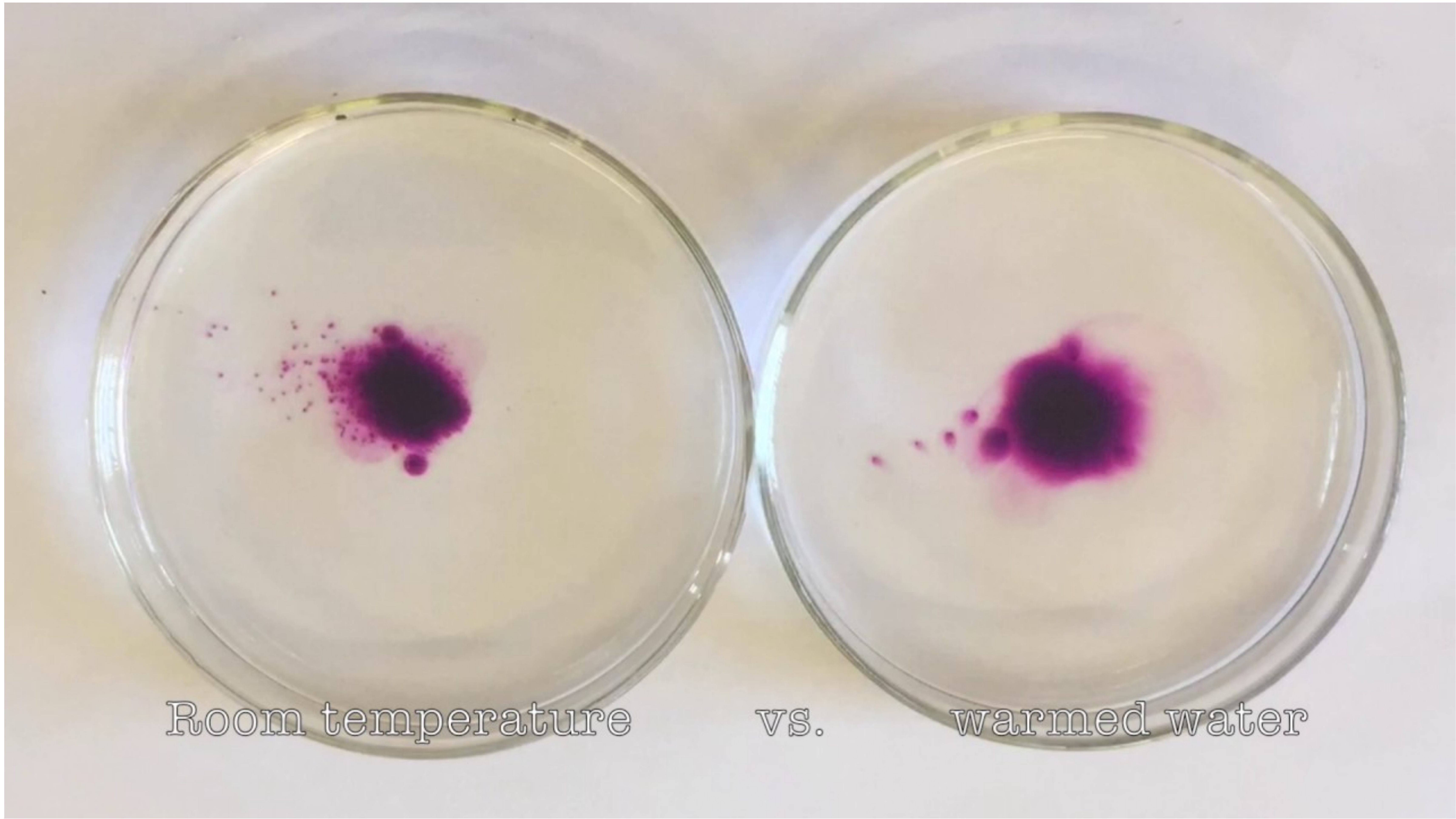
$$P(x_1 \to x_2) = P(x_2 \to x_1) \equiv P$$

- Example: $P = 0.5$

concentration gradients
lead to diffusion



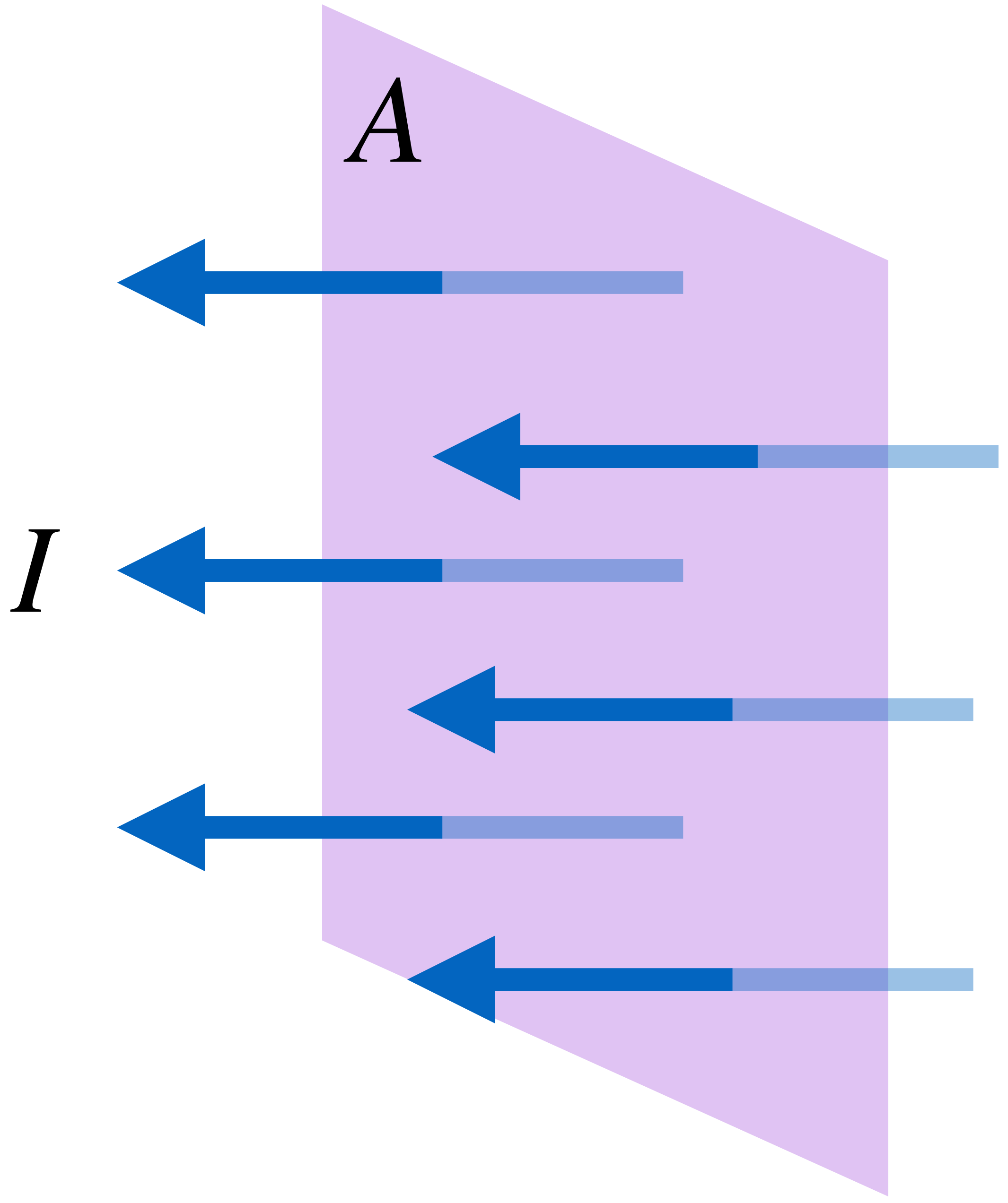
Random walk and diffusion



Diffusion flux

- Diffusion flux: number of ions crossing unit area in unit time

$$J = \frac{I}{A}$$



J : diffusion flux $[\mathrm{m^{-2}s^{-1}}]$

I : current of ions $[\mathrm{s^{-1}}]$

A : area $[\mathrm{m^2}]$

Fick's first law of diffusion

$$J_x = -D\frac{\partial n}{\partial x}$$

J_x : diffusion flux $[\mathrm{m^{-2}s^{-1}}]$

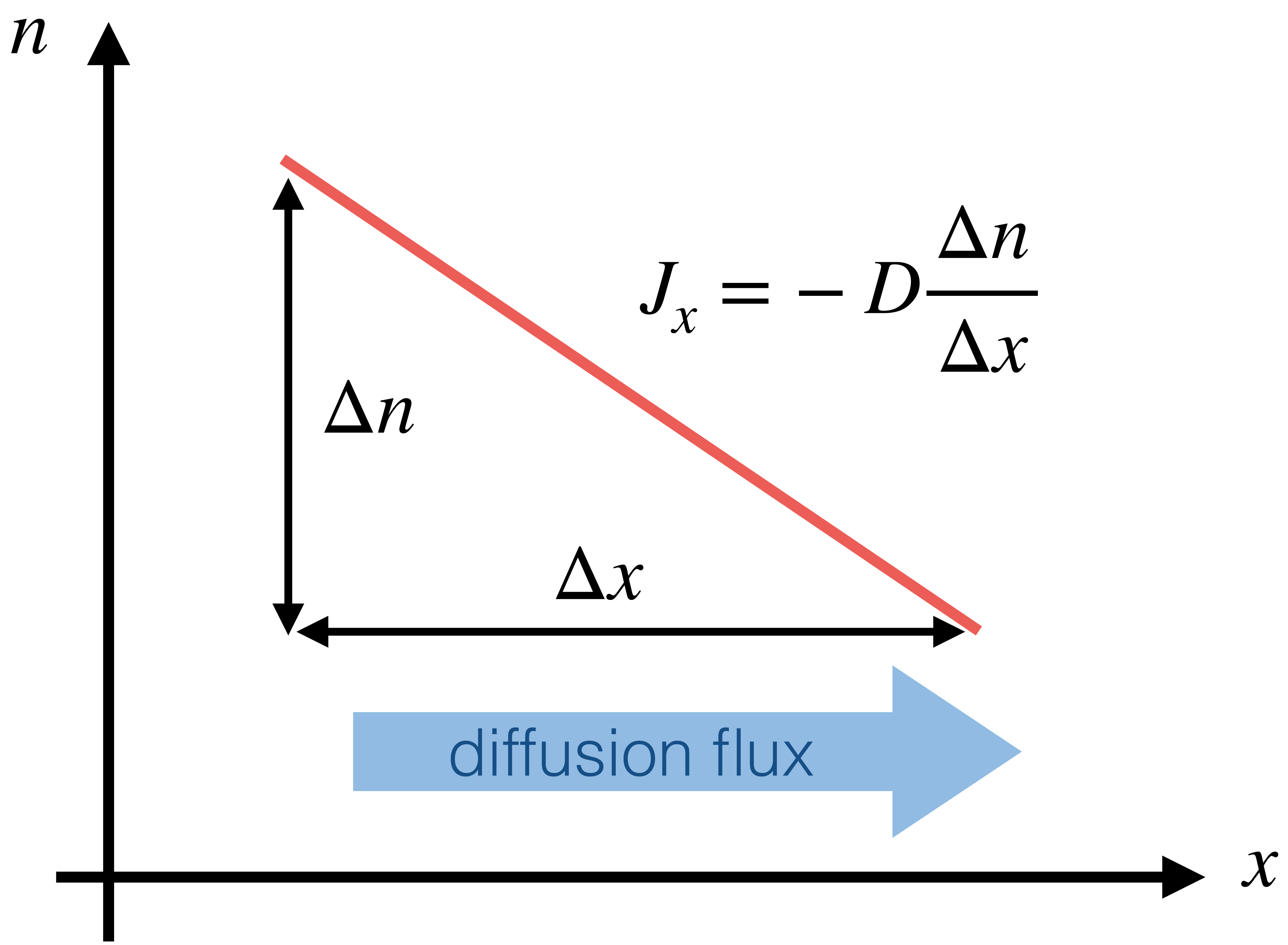
D : diffusion coefficient or diffusivity $[\mathrm{m^2s^{-1}}]$

n : number density of diffusing particles (concentration) $[\mathrm{m^{-3}}]$

x : position $[\mathrm{m}]$

Fick's first law of diffusion

$$J_x = -D\frac{\partial n}{\partial x}$$



Current density

$$J_x = -D\frac{\partial n}{\partial x}$$

- Ion of charge q:

$$j_x = -qD\frac{\partial n}{\partial x}$$

j_x : current density $[\mathrm{A\,m^{-2}}]$

q : ionic charge $[\mathrm{C}]$

D : diffusion coefficient or diffusivity $[\mathrm{m^2 s^{-1}}]$

n : number density of diffusing particles (concentration) $[\mathrm{m^{-3}}]$

x : position $[\mathrm{m}]$

Aside: Fick's second law of diffusion

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}$$

n : number density of diffusing particles (concentration) $[\mathrm{m}^{-3}]$

D : diffusion coefficient or diffusivity $[\mathrm{m}^2\mathrm{s}^{-1}]$

t : time $[\mathrm{s}]$

x : position $[\mathrm{m}]$

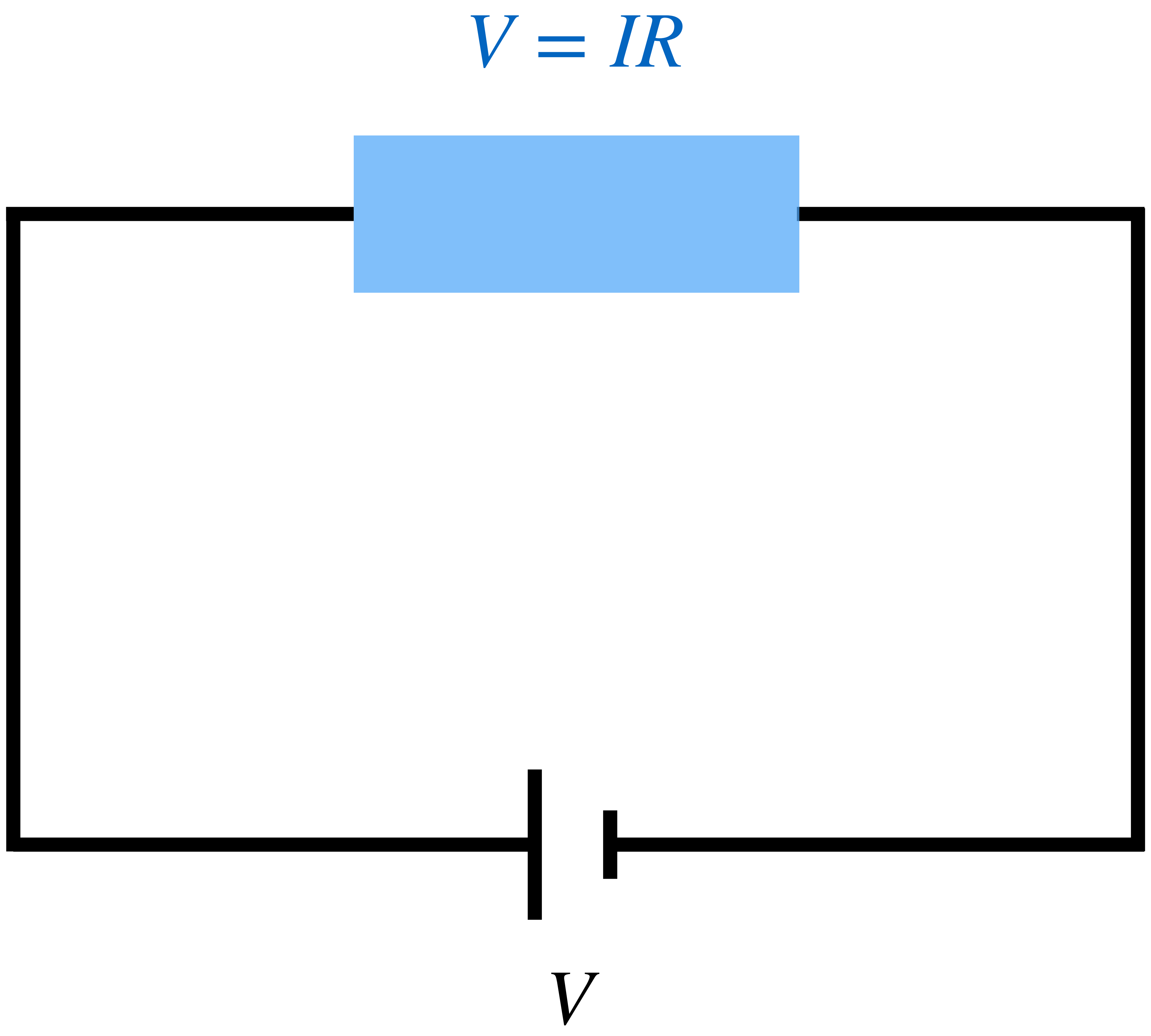
Ohm's law

$$I = \frac{V}{R}$$

I : current [A]

V : voltage [V]

R : resistance [Ω]



Ohm's law

$$I = \frac{V}{R}$$

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$

I : current $[\mathrm{A}]$

V : voltage $[\mathrm{V}]$

R : resistance $[\Omega]$

$\boldsymbol{j}$: current density $[\mathrm{Am^{-2}}]$

$\boldsymbol{E}$: electric field $[\mathrm{Vm^{-1}}]$

σ : conductivity $[\mathrm{Sm^{-1}}]$

Ohm's law

$$I = \frac{V}{R}$$

$$j_x = \sigma E_x = -\sigma \frac{\partial V}{\partial x}$$

I : current $[\mathrm{A}]$

V : voltage $[\mathrm{V}]$

R : resistance $[\Omega]$

j_x : current density $[\mathrm{Am}^{-2}]$

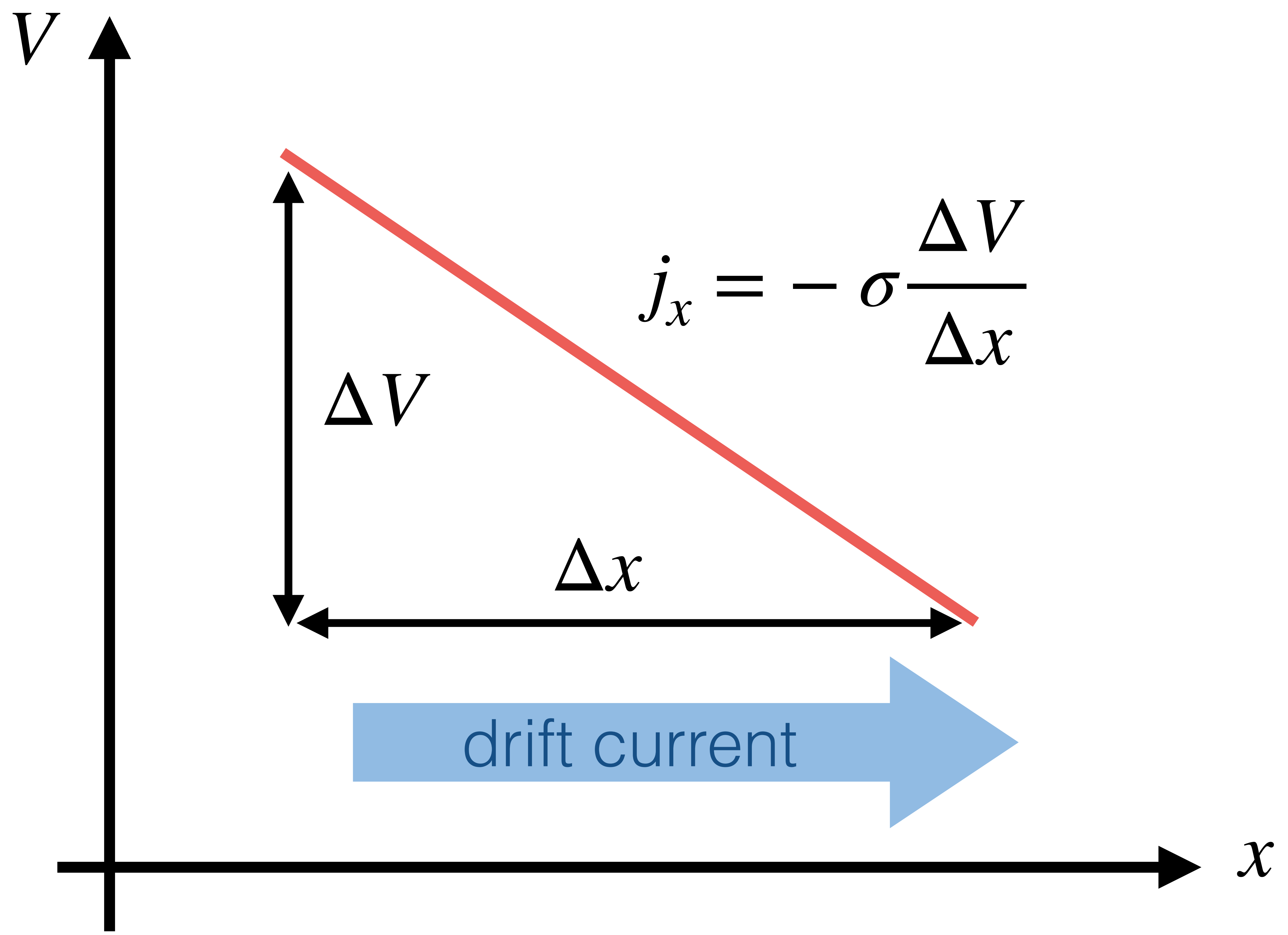
E_x : electric field $[\mathrm{Vm}^{-1}]$

σ : conductivity $[\mathrm{Sm}^{-1}]$

V : voltage $[\mathrm{V}]$

x : position $[\mathrm{m}]$

Drift current



$$j_x = \sigma E_x = -\sigma \frac{\partial V}{\partial x}$$

j_x : current density $[\mathrm{Am^{-2}}]$

E_x : electric field $[\mathrm{Vm^{-1}}]$

σ : conductivity $[\mathrm{Sm^{-1}}]$

V : voltage $[\mathrm{V}]$

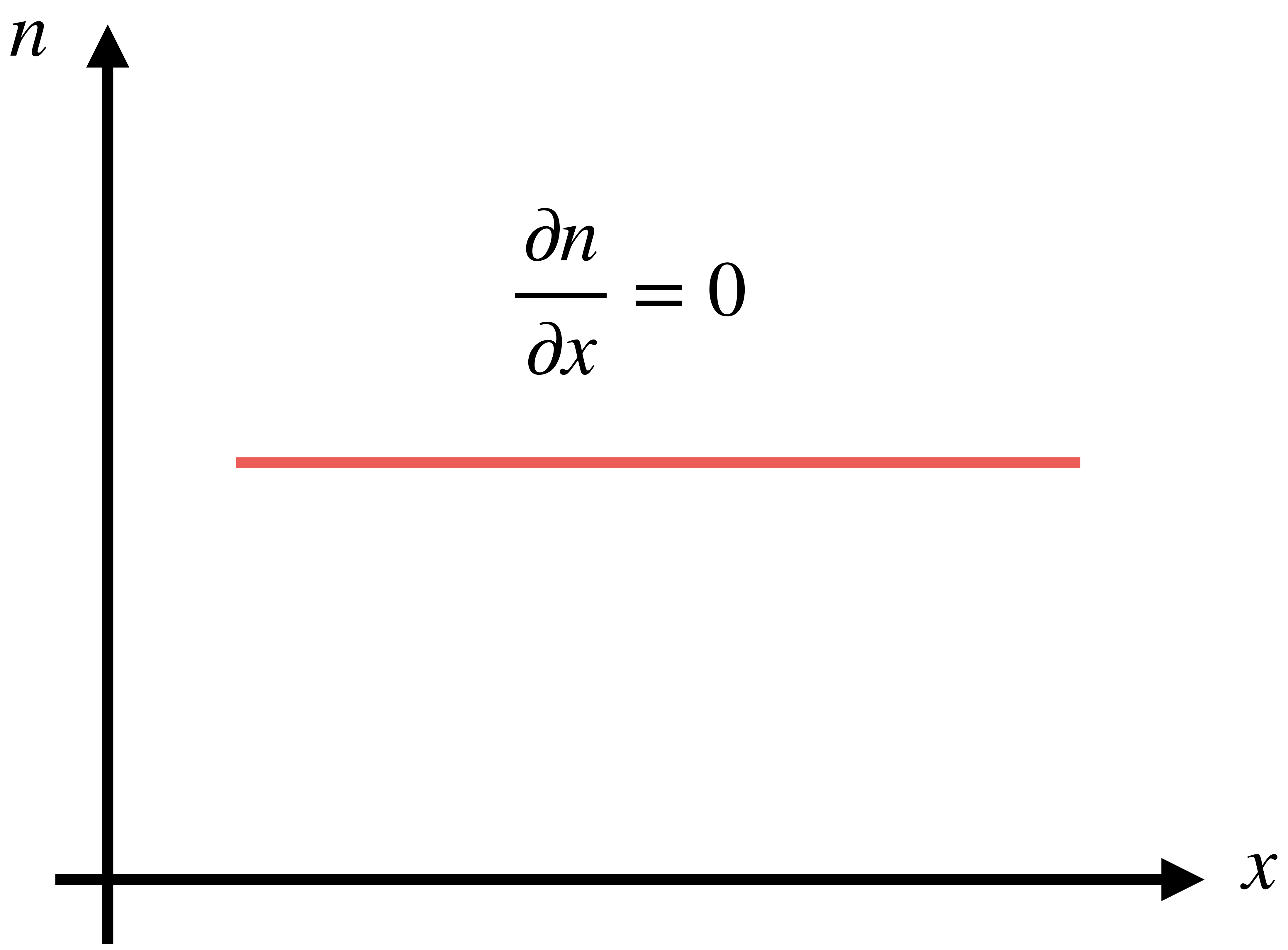
x : position $[\mathrm{m}]$

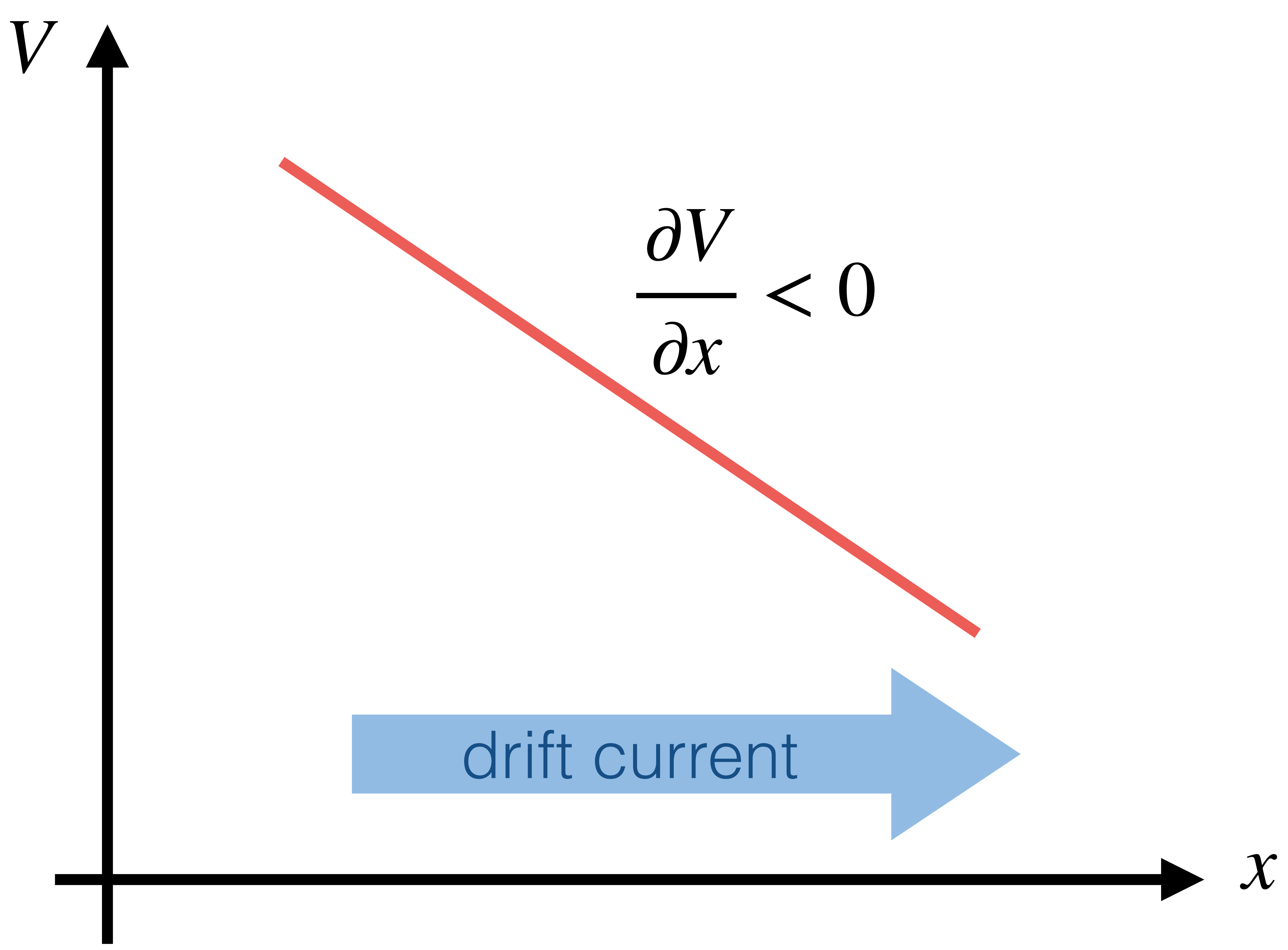
Fick's equation with drift current

$$j_x = -qD\frac{\partial n}{\partial x} - \sigma\frac{\partial V}{\partial x}$$

Fick's equation with drift current: steady state

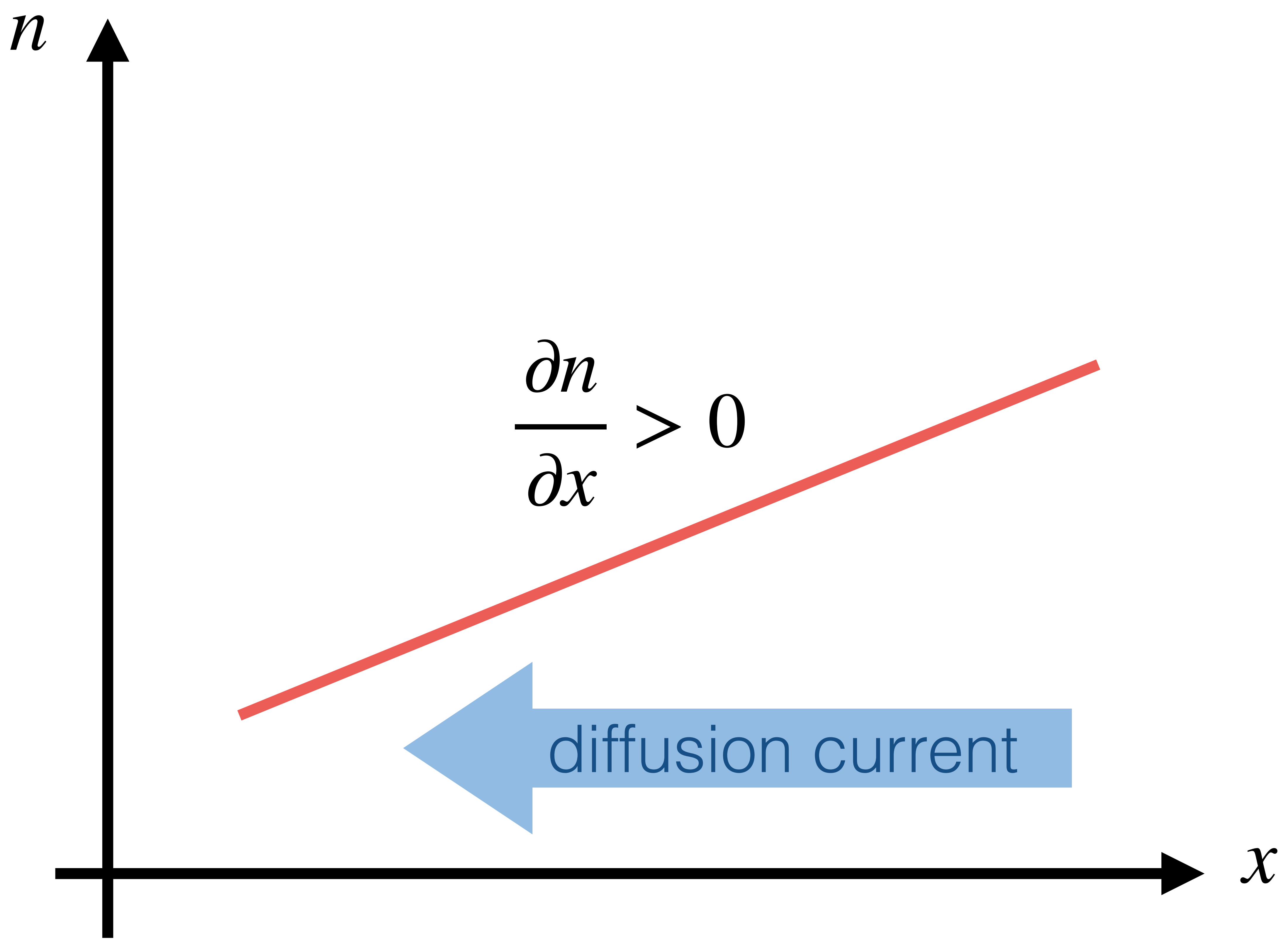
$$j_x = -qD\frac{\partial n}{\partial x} - \sigma\frac{\partial V}{\partial x}$$

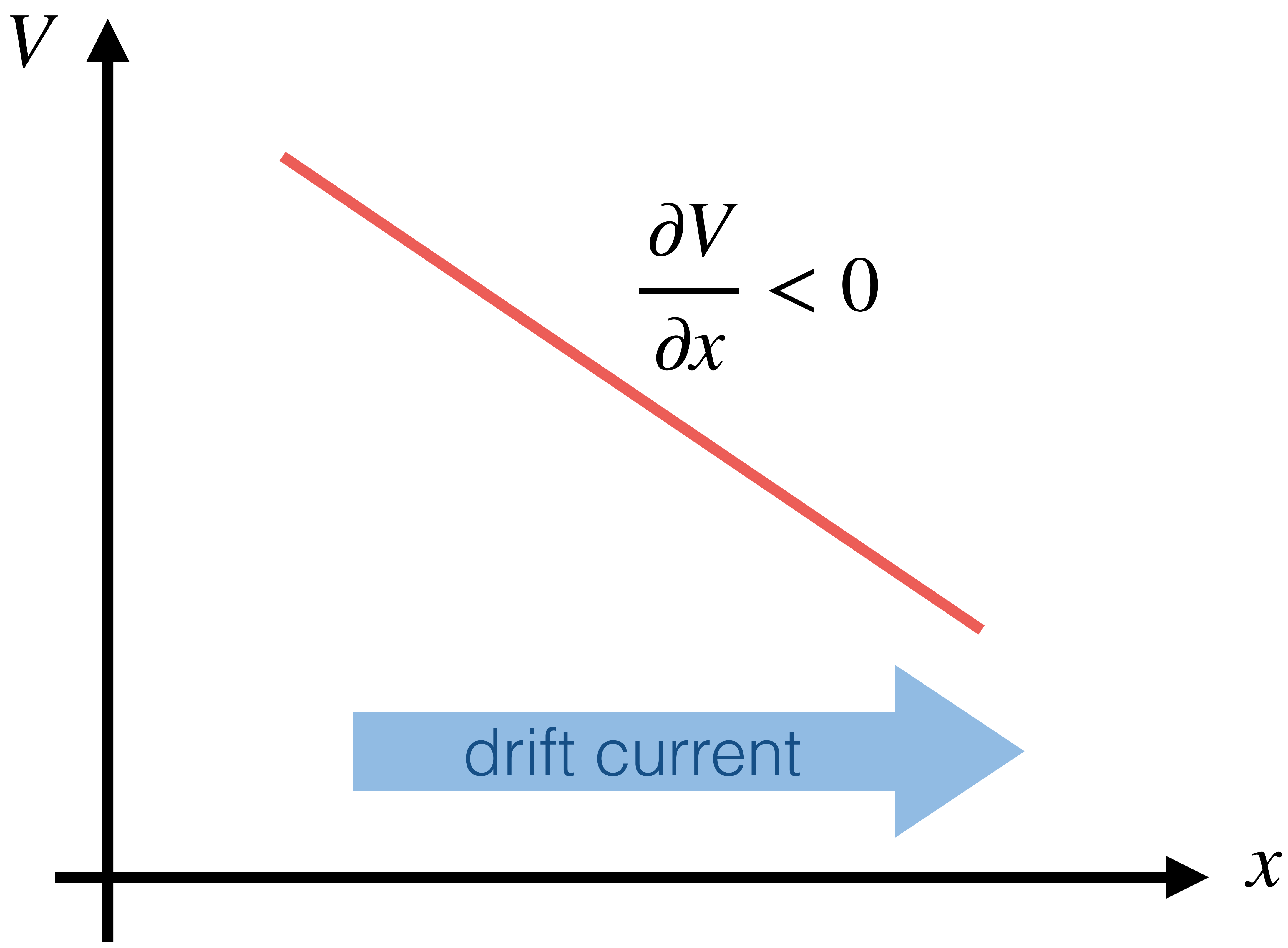




Fick's equation with drift current: steady state

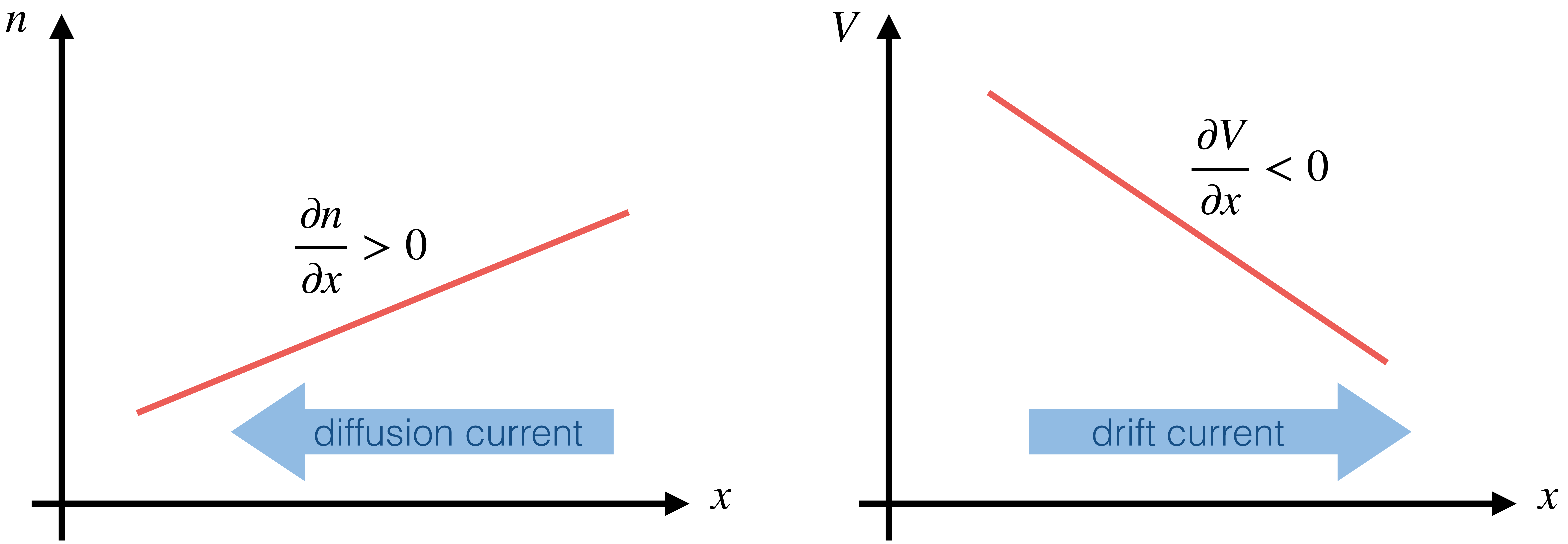
$$j_x = -qD\frac{\partial n}{\partial x} - \sigma\frac{\partial V}{\partial x}$$





Fick's equation with drift current: steady state

$$j_x = -qD\frac{\partial n}{\partial x} - \sigma\frac{\partial V}{\partial x} = 0$$



Nernst-Einstein equation

- *Derivation of Nernst-Einstein equation in Problem Set 3*

$$\frac{\sigma}{D} = \frac{nq^2}{k_\mathrm{B}T}$$